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Laplace operators on differential forms over configuration spaces

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Abstract

Spaces of differential forms over configuration spaces with Poisson measures are constructed. The corresponding Laplacians (of Bochner and de Rham type) on forms and associated semigroups are considered. Their probabilistic interpretation is given. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

Stochastic differential geometry of infinite-dimensional manifolds has been a very active topic of research in recent times. One of the important and intriguing problems discussed concerns the construction of spaces of differential forms over such manifolds and the study of the corresponding Laplace operators and associated (stochastic) cohomologies. A central role in this framework is played by the concept of the Dirichlet operator of a differentiable measure, which is actually an infinite-dimensional generalization of the Laplace–Beltrami operator on functions, respectively, the Laplace–Witten–de Rham operator on differential forms. The study of the latter operator and the associated semigroup on finite-dimensional manifolds was the subject of many works, and it leads to deep results on the interface

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of stochastic analysis, differential geometry and topology, and mathematical physics (see, e.g., [16,17,20,21,35]). Dirichlet forms and processes in connection with noncommutative C^*-algebras were considered in, e.g. [4,18,23].

The interest in the infinite-dimensional case is motivated by relations with supersymmetric quantum field theory. de Rham type operators acting on differential forms over Hilbert spaces were considered in [5,11–13]. In this relation, the mostly discussed example of an infinite-dimensional nonflat space is the loop space of a compact manifold (see [25,27,37]). Another important example given by the infinite product of compact manifolds was discussed in [1,2,14].

At the same time, there is a growing interest in geometry and analysis on Poisson spaces, i.e., on spaces of locally finite configurations in noncompact manifolds equipped with the Poisson measure. In [6–8], an approach to these spaces as to infinite-dimensional manifolds was initiated. This approach is motivated by the connection of such spaces with the theory of representations of diffeomorphism groups, see [22,24,36] (these references and [8,10] also contain discussion of relations with quantum physics). In fact, the configuration space, which does not possess the structure of a smooth manifold in the proper sense, can be equipped with some "Riemannian-like" structure generated by the action of the diffeomorphism group of the initial manifold. We refer the reader to [9,10,33], and references therein for further discussion of analysis on Poisson spaces and applications.

In the present work, we develop this point of view. We define spaces of differential forms over Poisson spaces and study Laplace operators acting in these spaces. We show, in particular, that the corresponding de Rham Laplacian can be expressed in terms of the Dirichlet operator on functions on the Poisson space and the Witten Laplacian on the initial manifold associated with the intensity of the corresponding Poisson measure. We give a probabilistic interpretation and investigate some properties of the associated semigroups. The main general aim of our approach is to develop a framework which extends to Poisson spaces (as infinite-dimensional manifolds), the finite-dimensional Hodge–de Rham theory.

The results of the present paper in the special case of 1-forms were presented in [3]. A different approach to the construction of differential forms and related objects over Poisson spaces, based on the "transfer principle" from Wiener spaces, is proposed in [30] (see also [28] and [29]).

2. Differential forms over configuration spaces

The aim of this section is to define differential forms over configuration spaces (as infinite-dimensional manifold). First, we recall some known facts and definitions concerning "manifold-like" structures and functional calculus on these spaces.

2.1. Functional calculus on configuration spaces

Our presentation in this section is based upon [8], however, for later use in the present paper we give a different description of some objects and results occurring in [8].

Let X be a complete, connected, oriented, C^∞ (noncompact) Riemannian manifold of dimension d. We denote by $\langle\bullet,\bullet\rangle_x$ the corresponding inner product in the tangent space T_xX to X at a point $x\in X$. The associated norm will be denoted by $|\bullet|_x$. Let also ∇^X stand for the gradient on X.

The configuration space Γ_X over X is defined as the set of all locally finite subsets (configurations) in X:

$$\Gamma_X := \{\gamma\subset X \,|\, |\gamma\cap\Lambda|<\infty \text{ for each compact } \Lambda\subset X\}.$$

Here, $|A|$ denotes the cardinality of the set A.

We can identify any $\gamma\in\Gamma_X$ with the positive integer-valued Radon measure

$$\sum_{x\in\gamma}\varepsilon_x \in \mathcal{M}(X),$$

where ε_x is the Dirac measure with mass at x, $\sum_{x\in\varnothing}\varepsilon_x$:= zero measure, and $\mathcal{M}(X)$ denotes the set of all positive Radon measures on the Borel σ-algebra $\mathcal{B}(X)$. The space Γ_X is endowed with the relative topology as a subset of the space $\mathcal{M}(X)$ with the vague topology, i.e., the weakest topology on Γ_X such that all maps

$$\Gamma_X \ni \gamma \mapsto \langle f,\gamma\rangle := \int_X f(x)\gamma(\mathrm{d}x) \equiv \sum_{x\in\gamma} f(x)$$

are continuous. Here, $f\in C_0(X)$ (:= the set of all continuous functions on X with compact support). Let $\mathcal{B}(\Gamma_X)$ denote the corresponding Borel σ-algebra.

Following [8], we define the tangent space to Γ_X at a point γ as the Hilbert space

$$T_\gamma\Gamma_X := L^2(X\to TX;\mathrm{d}\gamma),$$

or equivalently

$$T_\gamma\Gamma_X = \bigoplus_{x\in\gamma} T_xX. \tag{2.1}$$

(Compare also with [36, Appendix 3].) The scalar product and the norm in $T_\gamma\Gamma_X$ will be denoted by $\langle\bullet,\bullet\rangle_\gamma$ and $\|\bullet\|_\gamma$, respectively. Thus, each $V(\gamma)\in T_\gamma\Gamma_X$ has the form $V(\gamma)=(V(\gamma)_x)_{x\in\gamma}$, where $V(\gamma)_x\in T_xX$, and

$$\|V(\gamma)\|_\gamma^2 = \sum_{x\in\gamma}|V(\gamma)_x|_x^2.$$

Let $\gamma\in\Gamma_X$ and $x\in\gamma$. By $\mathcal{O}_{\gamma,x}$, we will denote an arbitrary open neighborhood of x in X such that the intersection of the closure of $\mathcal{O}_{\gamma,x}$ in X with $\gamma\setminus\{x\}$ is the empty set. For any fixed finite subconfiguration $\{x_1,\dots,x_k\}\subset\gamma$, we will always consider open neighborhoods $\mathcal{O}_{\gamma,x_1},\dots,\mathcal{O}_{\gamma,x_k}$ with disjoint closures.

Now, for a measurable function $F:\Gamma_X\to\mathbb{R}$, $\gamma\in\Gamma_X$, and $\{x_1,\dots,x_k\}\subset\gamma$, we define a function $F_{x_1,\dots,x_k}(\gamma,\bullet):\mathcal{O}_{\gamma,x_1}\times\cdots\times\mathcal{O}_{\gamma,x_k}\to\mathbb{R}$ by

$$\begin{aligned}\mathcal{O}_{\gamma,x_1}\times\cdots\times\mathcal{O}_{\gamma,x_k} \ni (y_1,\dots,y_k) &\mapsto F_{x_1,\dots,x_k}(\gamma,y_1,\dots,y_k)\\ &:= F((\gamma\setminus\{x_1,\dots,x_k\})\cup\{y_1,\dots,y_k\})\in\mathbb{R}.\end{aligned}$$

Since we will be interested only in the local behavior of the function $F_{x_1,\dots,x_k}(\gamma,\bullet)$ around the point $(x_1,\dots,x_k)$, we will not write explicitly which neighborhoods $\mathcal{O}_{\gamma,x_i}$ we use.

Definition 2.1. We say that a function $F:\Gamma_X\to\mathbb{R}$ is differentiable at $\gamma\in\Gamma_X$ if for each $x\in\gamma$ the function $F_x(\gamma,\bullet)$ is differentiable at x and

$$\nabla^\Gamma F(\gamma)=(\nabla^\Gamma F(\gamma)_x)_{x\in\gamma}\in T_\gamma\Gamma_X,$$

where

$$\nabla^\Gamma F(\gamma)_x:=\nabla^X F_x(\gamma,x).$$

We will call $\nabla^\Gamma F(\gamma)$ the *gradient* of F at γ.

For a function F differentiable at γ and a vector $V(\gamma)\in T_\gamma\Gamma_X$, the *directional derivative* of F at the point γ along $V(\gamma)$ is defined by

$$\nabla^\Gamma_V F(\gamma):=\langle\nabla^\Gamma F(\gamma),V(\gamma)\rangle_\gamma.$$

In what follows, we will also use the shorthand notation

$$\nabla^X_x F(\gamma):=\nabla^X F_x(\gamma,x), \tag{2.2}$$

so that

$$\nabla^\Gamma F(\gamma)=(\nabla^X_x F(\gamma))_{x\in\gamma}.$$

It is easy to see that the operation ∇^Γ satisfies the usual properties of differentiation, including the Leibniz rule. We define a class $\mathcal{FC}$ of smooth cylinder functions on Γ_X as follows.

Definition 2.2. A measurable bounded function $F:\Gamma_X\to\mathbb{R}$ belongs to $\mathcal{FC}$ iff:
1. there exists a compact $\Lambda\subset X$ such that $F(\gamma)=F(\gamma_\Lambda)$ for all $\gamma\in\Gamma_X$, where $\gamma_\Lambda:=\gamma\cap\Lambda$;
2. for any $\gamma\in\Gamma_X$ and $\{x_1,\dots,x_k\}\subset\gamma$, $k\in\mathbb{N}$, the function $F_{x_1,\dots,x_k}(\gamma,\bullet)$ is infinitely differentiable with partial derivatives uniformly bounded in γ and $x_1,\dots,x_k$ (i.e., the majorizing constant depends only on the order of differentiation but not on the specific choice of $\gamma\in\Gamma_X$, $k\in\mathbb{N}$, and $\{x_1,\dots,x_k\}\subset\gamma$).

Let us note that, for $F\in\mathcal{FC}$, only a finite number of coordinates of $\nabla^\Gamma F(\gamma)$ are not equal to zero, and so $\nabla^\Gamma F(\gamma)\in T_\gamma\Gamma_X$. Thus, each $F\in\mathcal{FC}$ is differentiable at any point $\gamma\in\Gamma_X$ in the sense of Definition 2.1.

Remark 2.1. *In* [8], *the authors introduced the class $\mathcal{FC}^\infty_{\mathrm b}(\mathcal{D},\Gamma_X)$ of functions on Γ_X of the form*

$$F(\gamma)=g_F(\langle\varphi_1,\gamma\rangle,\dots,\langle\varphi_N,\gamma\rangle), \tag{2.3}$$

where $g_F \in C_{\mathrm{b}}^{\infty}(\mathbb{R}^N)$ *and* $\varphi_1, \dots, \varphi_N \in \mathcal{D} := C_0^{\infty}(X)$ (:= *the set of all* C^{∞}*-functions on* X *with compact support*). *Evidently, we have the inclusion*

$$\mathcal{F}\mathcal{C}_{\mathrm{b}}^{\infty}(\mathcal{D}, \Gamma_X) \subset \mathcal{F}\mathcal{C},$$

and moreover, the gradient of F of the form (2.3) *in the sense of Definition* 2.1,

$$\nabla^{\Gamma} F(\gamma)_x = \sum_{i=1}^{N} \frac{\partial g_F}{\partial s_i} (\langle \varphi_1, \gamma \rangle, \dots, \langle \varphi_N, \gamma \rangle) \nabla^X \varphi_i(x),$$

coincides with the gradient of this function in the sense of [8].

2.2. Tensor bundles and cylinder forms over configuration spaces

Our next aim is to introduce differential forms on Γ_X. Vector fields and first-order differential forms on Γ_X will be identified with sections of the bundle $T\Gamma_X$. Higher order differential forms will be identified with sections of tensor bundles $\wedge^n(T\Gamma_X)$ with fibers

$$\wedge^n(T_\gamma \Gamma_X) := \wedge^n(L^2(X \to TX; \gamma)),$$

where $\wedge^n(\mathcal{H})$ (or $\mathcal{H}^{\wedge n}$) stands for the nth antisymmetric tensor power of a Hilbert space $\mathcal{H}$. In what follows, we will use different representations of this space. Because of (2.1), we have

$$\wedge^n(T_\gamma \Gamma_X) = \wedge^n \left(\bigoplus_{x \in \gamma} T_x X \right). \tag{2.4}$$

Let us introduce the factor space X^n/S_n, where S_n is the permutation group of $\{1, \dots, n\}$ which naturally acts on X^n:

$$\sigma(x_1, \dots, x_n) = (x_{\sigma(1)}, \dots, x_{\sigma(n)}), \quad \sigma \in S_n.$$

The space X^n/S_n consists of equivalence classes $[x_1, \dots, x_n]$ and we will denote by $[x_1, \dots, x_n]_d$ an equivalence class $[x_1, \dots, x_n]$ such that the equality $x_{i_1} = x_{i_2} = \cdots = x_{i_k}$ can hold only for $k \le d$ points. (In other words, any equivalence class $[x_1, \dots, x_n]$ is a multiple configuration in X, while $[x_1, \dots, x_n]_d$ is a multiple configuration with multiplicity of points $\le d$.) In what follows, instead of writing $[x_1, \dots, x_n]_d : \{x_1, \dots, x_n\} \subset \gamma$, we will use the shortened notation $[x_1, \dots, x_n]_d \subset \gamma$, though $[x_1, \dots, x_n]_d$ is not, of course, a set. We then have from (2.4),

$$\wedge^n(T_\gamma \Gamma_X) = \bigoplus_{[x_1, \dots, x_n]_d \subset \gamma} T_{x_1} X \wedge T_{x_2} X \wedge \cdots \wedge T_{x_n} X. \tag{2.5}$$

Here, the space $T_{x_1} X \wedge T_{x_2} X \wedge \cdots \wedge T_{x_n} X$ is understood as a subspace of the Hilbert space $(T_{y_1} X \oplus T_{y_2} X \oplus \cdots \oplus T_{y_k} X)^{\otimes n}$, where $\{y_1, \dots, y_k\}$ is the set of the different x_j's, $j = 1, \dots, n$. To see that (2.5) holds, notice that

$$(T_{y_1} X \oplus T_{y_2} X \oplus \cdots \oplus T_{y_k} X)^{\otimes n} \simeq (T_{y_{\nu(1)}} X \oplus T_{y_{\nu(2)}} X \oplus \cdots \oplus T_{y_{\nu(k)}} X)^{\otimes n}, \quad \nu \in S_k, \tag{2.6}$$

where "$\simeq$" means isomorphism, and moreover $T_{x_1}X \wedge T_{x_2}X \wedge \cdots \wedge T_{x_n}X$ and $T_{x_{\sigma(1)}}X \wedge T_{x_{\sigma(2)}}X \wedge \cdots \wedge T_{x_{\sigma(n)}}X$, $\sigma \in S_n$, coincide as subspaces of the space (2.6).

Thus, under a differential form W of order n, $n \in \mathbb{N}$, over Γ_X, we will understand a mapping

$$\Gamma_X \ni \gamma \mapsto W(\gamma) \in \wedge^n(T_\gamma \Gamma_X). \tag{2.7}$$

We denote by $W(\gamma)_{[x_1,\dots,x_n]_d}$ the corresponding component of $W(\gamma)$ in the decomposition (2.5).

In particular, in the case $n = 1$, a 1-form V over Γ_X is given by the mapping

$$\Gamma_X \ni \gamma \mapsto V(\gamma) = (V(\gamma)_x)_{x\in\gamma} \in T_\gamma \Gamma_X.$$

For fixed $\gamma \in \Gamma_X$ and $x \in \gamma$, we consider the mapping

$$\mathcal{O}_{\gamma,x} \ni y \mapsto W_x(\gamma, y) := W(\gamma_y) \in \wedge^n(T_{\gamma_y}\Gamma_X),$$

where $\gamma_y := (\gamma \setminus \{x\}) \cup \{y\}$, which is a section of the Hilbert bundle

$$\wedge^n(T_{\gamma_y}\Gamma_X) \mapsto y \in \mathcal{O}_{\gamma,x} \tag{2.8}$$

over $\mathcal{O}_{\gamma,x}$. The Levi–Civita connection on *TX* generates in a natural way a "product" connection on this bundle. We denote by $\nabla^X_{\gamma,x}$ the corresponding covariant derivative, and use the notation

$$\nabla^X_x W(\gamma) := \nabla^X_{\gamma,x} W_x(\gamma, x) \in T_x X \otimes (\wedge^n(T_\gamma \Gamma_X))$$

if the section $W_x(\gamma, \bullet)$ is differentiable at x. Analogously, we denote by Δ^X_x the corresponding Bochner Laplacian associated with the volume measure m on $\mathcal{O}_{\gamma,x}$ (see Section 3.2 where the notion of Bochner Laplacian is recalled).

Similarly, for a fixed $\gamma \in \Gamma_X$ and $\{x_1, \dots, x_k\} \subset \gamma$, we define a mapping

$$\begin{aligned}\mathcal{O}_{\gamma,x_1} \times \cdots \times \mathcal{O}_{\gamma,x_k} &\ni (y_1, \dots, y_k) \mapsto W_{x_1,\dots,x_k}(\gamma, y_1, \dots, y_k)\\ &:= W(\gamma_{y_1,\dots,y_k}) \in \wedge^n(T_{\gamma_{y_1,\dots,y_k}}\Gamma_X),\end{aligned}$$

where $\gamma_{y_1,\dots,y_k} := (\gamma \setminus \{x_1, \dots, x_k\}) \cup \{y_1, \dots, y_k\}$, which is a section of the Hilbert bundle

$$\wedge^n(T_{\gamma_{y_1,\dots,y_k}}\Gamma_X) \mapsto (y_1, \dots, y_k) \in \mathcal{O}_{\gamma,x_1} \times \cdots \times \mathcal{O}_{\gamma,x_k} \tag{2.9}$$

over $\mathcal{O}_{\gamma,x_1} \times \cdots \times \mathcal{O}_{\gamma,x_k}$.

Let us remark that, for any $\eta \subset \gamma$, the space $\wedge^n(T_\eta \Gamma_X)$ can be identified in a natural way with a subspace of $\wedge^n(T_\gamma \Gamma_X)$. In this sense, we will use expressions of the type $W(\gamma) = W(\eta)$ without additional explanations. A set $\mathcal{F}\Omega^n$ of smooth cylinder n-forms over Γ_X will be defined as follows.

Definition 2.3. $\mathcal{F}\Omega^n$ is the set of n-forms W over Γ_X which satisfy the following conditions:

1. there exists a compact $\Lambda = \Lambda(W) \subset X$ such that $W(\gamma) = W(\gamma_\Lambda)$;

2. for each $\gamma \in \Gamma_X$ and $\{x_1, \dots, x_k\} \subset \gamma$, the section $W_{x_1,\dots,x_n}(\gamma, \bullet)$ of the bundle (2.9) is infinitely differentiable at $(x_1, \dots, x_k)$, and bounded together with partial derivatives component-wise in the sense of decomposition (2.5), uniformly in $\gamma, x_1, \dots, x_k$, and the component.

Remark 2.2. *For each $W \in \mathcal{F}\Omega^n$, $\gamma \in \Gamma_X$, and any open bounded $\Lambda \supset \Lambda(W)$, we can define the form $W_{\Lambda,\gamma}$ on $\mathcal{O}_{\gamma,x_1} \times \cdots \times \mathcal{O}_{\gamma,x_k}$ by*

$$W_{\Lambda,\gamma}(y_1, \dots, y_k) = \mathrm{Proj}_{\wedge^n(T_{y_1}X\oplus\cdots\oplus T_{y_k}X)} W((\gamma \setminus \{x_1, \dots, x_k\}) \cup \{y_1, \dots, y_k\}), \tag{2.10}$$

where $\{x_1, \dots, x_k\} = \gamma \cap \Lambda$. The item (2) *of Definition* 2.3 *is obviously equivalent to the assumption that $W_{\Lambda,\gamma}$ is smooth and bounded together with all partial derivatives component-wise uniformly in γ (for some Λ and consequently for any $\Lambda \supset \Lambda(W)$).*

Definition 2.4. We define the covariant derivative $\nabla^\Gamma W$ of a form W given by (2.7) as the mapping

$$\Gamma_X \ni \gamma \mapsto \nabla^\Gamma W(\gamma) := (\nabla_x^X W(\gamma))_{x\in\gamma} \in T_\gamma\Gamma_X \otimes (\wedge^n(T_\gamma\Gamma_X))$$

if for all $\gamma \in \Gamma_X$ and $x \in \gamma$ the form $W_x(\gamma, \bullet)$ is differentiable at x and the $\nabla^\Gamma W(\gamma)$ just defined indeed belongs to $T_\gamma\Gamma_X \otimes (\wedge^n(T_\gamma\Gamma_X))$.

Remark 2.3. *For each $W \in \mathcal{F}\Omega^n$, the covariant derivative $\nabla^\Gamma W$ exists, and moreover only a finite number of the coordinates $\nabla^\Gamma W(\gamma)_{x,[x_1,\dots,x_n]_d}$ in the decomposition*

$$T_\gamma\Gamma_X \otimes (\wedge^n(T_\gamma\Gamma_X)) = \bigoplus_{x\in\gamma,\,[x_1,\dots,x_n]_d\subset\gamma} T_xX \otimes (T_{x_1}X \wedge \cdots \wedge T_{x_n}X)$$

are not equal to zero.

Remark 2.4. *For each $W \in \mathcal{F}\Omega^n$, $\gamma \in \Gamma_X$, $x \in \gamma$, and $[x_1, \dots, x_n]_d \subset \gamma$, we define the mapping $W_x(\gamma, \bullet)_{[x_1,\dots,x_n]_d}$ as follows: if $x \neq x_j$ for all $j = 1, \dots, n$, then*

$$\begin{aligned}\mathcal{O}_{\gamma,x} \ni y \mapsto\ & W_x(\gamma, y)_{[x_1,\dots,x_n]_d} \\ &:= W((\gamma \setminus \{x\}) \cup \{y\})_{[x_1,\dots,x_n]_d} \in T_{x_1}X \wedge \cdots \wedge T_{x_n}X,\end{aligned}$$

and if $x = x_i$ for some $x_i \in \{x_1, \dots, x_n\}$, then

$$\begin{aligned}\mathcal{O}_{\gamma,x} \ni y \mapsto\ & W_x(\gamma, y)_{[x_1,\dots,x_n]_d} \\ &:= W((\gamma \setminus \{x\}) \cup \{y\})_{[y_1,\dots,y_n]_d} \in T_{y_1}X \wedge \cdots \wedge T_{y_n}X,\end{aligned}$$

where $y_j = x_j$ if $x \neq x_j$ and $y_j = y$ otherwise. Then, the condition (2) *of Definition* 2.3 *yields, in particular, that the mapping $W_x(\gamma, \bullet)_{[x_1,\dots,x_n]_d}$ is C^∞ for all $x \in \gamma$ and $[x_1, \dots, x_n]_d \subset \gamma$. Now, we have*

$$\nabla^\Gamma W(\gamma)_{x,\,[x_1,\dots,x_n]_d} = \nabla_x^X W(\gamma)_{[x_1,\dots,x_n]_d},$$

where

$$\nabla_x^X W(\gamma)_{[x_1,\dots,x_n]_d} := \nabla^X W_x(\gamma, x)_{[x_1,\dots,x_n]_d}.$$

Notice that, in the case where $x \neq x_j$ for all $j = 1, \dots, n$, $\nabla^X W_x(\gamma, \bullet)_{[x_1,\dots,x_n]_d}$ means, in fact, the usual derivative of a mapping defined on $\mathcal{O}_{\gamma,x}$ and taking values in the fixed vector space $T_{x_1}X \wedge \cdots \wedge T_{x_n}X$. On the other hand, if x does coincide with some $x_i \in \{x_1, \dots, x_n\}$, then the expression $\nabla^X W_x(\gamma, x)_{[x_1,\dots,x_n]_d}$ can be understood as the $T_xX \otimes (T_{x_1}X \wedge \cdots \wedge T_{x_n}X)$-coordinate of the covariant derivative of the n-form

$$\begin{aligned}&\mathcal{O}_{\gamma,y_1} \times \cdots \times \mathcal{O}_{\gamma,y_k} \ni (z_1, \dots, z_k)\\ &\quad\mapsto \mathrm{Proj}_{\wedge^n(T_{z_1}X\oplus\cdots\oplus T_{z_k}X)} W((\gamma \setminus \{y_1, \dots, y_k\}) \cup \{z_1, \dots, z_k\})\end{aligned} \tag{2.11}$$

at the point $(y_1, \dots, y_k)$, where $\{y_1, \dots, y_k\}$ is the set of all the different x_j's, $j = 1, \dots, n$. In fact, the last sentence was just an alternative description of the notion of covariant derivative $\nabla^X W_x(\gamma, x)_{[x_1,\dots,x_n]_d}$ of the mapping $W_x(\gamma, \bullet)_{[x_1,\dots,x_n]_d}$ in the case where x coincides with some x_i.

Proposition 2.1. *For arbitrary $W^{(1)}, W^{(2)} \in \mathcal{F}\Omega^n$, we have*

$$\begin{aligned}\nabla^\Gamma \langle W^{(1)}(\gamma), W^{(2)}(\gamma)\rangle_{\wedge^n(T_\gamma\Gamma_X)} &= \langle \nabla^\Gamma W^{(1)}(\gamma), W^{(2)}(\gamma)\rangle_{\wedge^n(T_\gamma\Gamma_X)}\\ &\quad+\langle W^{(1)}(\gamma), \nabla^\Gamma W^{(2)}(\gamma)\rangle_{\wedge^n(T_\gamma\Gamma_X)}.\end{aligned}$$

Proof. We have, for any fixed $\gamma \in \Gamma_X$,

$$\begin{aligned}\nabla^\Gamma \langle W^{(1)}(\gamma), W^{(2)}(\gamma)\rangle_{\wedge^n(T_\gamma\Gamma_X)} &= \sum_{x\in\gamma} \nabla_x^X \langle W^{(1)}(\gamma), W^{(2)}(\gamma)\rangle_{\wedge^n(T_\gamma\Gamma_X)}\\ &= \sum_{x\in\gamma} \nabla_x^X \sum_{[x_1,\dots,x_n]_d\subset\gamma} \langle W^{(1)}(\gamma)_{[x_1,\dots,x_n]_d}, W^{(2)}(\gamma)_{[x_1,\dots,x_n]_d}\rangle_{T_{x_1}X\wedge\cdots\wedge T_{x_n}X}\\ &= \sum_{x\in\gamma} \sum_{[x_1,\dots,x_n]_d\subset\gamma} \nabla_x^X \langle W^{(1)}(\gamma)_{[x_1,\dots,x_n]_d}, W^{(2)}(\gamma)_{[x_1,\dots,x_n]_d}\rangle_{T_{x_1}X\wedge\cdots\wedge T_{x_n}X}\\ &= \sum_{x\in\gamma} \sum_{[x_1,\dots,x_n]_d\subset\gamma} [\langle \nabla_x^X W^{(1)}(\gamma)_{[x_1,\dots,x_n]_d}, W^{(2)}_{[x_1,\dots,x_n]_d}\rangle_{T_{x_1}X\wedge\cdots\wedge T_{x_n}X}\\ &\quad+\langle W^{(1)}(\gamma)_{[x_1,\dots,x_n]_d}, \nabla_x^X W^{(2)}(\gamma)_{[x_1,\dots,x_n]_d}\rangle_{T_{x_1}X\wedge\cdots\wedge T_{x_n}X}]\\ &= \langle \nabla^\Gamma W^{(1)}(\gamma), W^{(2)}(\gamma)\rangle_{\wedge^n(T_\gamma\Gamma_X)} + \langle W^{(1)}(\gamma), \nabla^\Gamma W^{(2)}(\gamma)\rangle_{\wedge^n(T_\gamma\Gamma_X)}.\end{aligned}$$

All the sums above are actually finite because of the definition of $\mathcal{F}\Omega^n$. □

2.3. Square-integrable n-forms

Our next goal is to give a description of the space of n-forms that are square-integrable with respect to the Poisson measure.

Let m be the volume measure on X, let $\rho : X \to \mathbb{R}$ be a measurable function such that $\rho > 0$ m-a.e., and $\rho^{1/2} \in H^{1,2}_{\mathrm{loc}}(X)$, and define the measure $\sigma(\mathrm{d}x) := \rho(x)\, m(\mathrm{d}x)$. Here,

$H^{1,2}_{\mathrm{loc}}(X)$ denotes the local Sobolev space of order 1 in $L^2_{\mathrm{loc}}(X; m)$. Then, σ is a nonatomic Radon measure on X.

Let π_σ stand for the Poisson measure on Γ_X with intensity σ. This measure is characterized by its Laplace transform

$$\int_{\Gamma_X} \mathrm{e}^{\langle f,\gamma\rangle}\pi_\sigma(\mathrm{d}\gamma) = \exp\int_X (\mathrm{e}^{f(x)}-1)\sigma(\mathrm{d}x), \quad f \in \mathcal{D}.$$

Let $F \in L^1(\Gamma_X; \pi_\sigma)$ be cylindrical, i.e., there exits a compact $\Lambda \subset X$ such that $F(\gamma) = F(\gamma_\Lambda)$. Then, one has the following formula, which we will use many times:

$$\int_{\Gamma_X} F(\gamma)\pi_\sigma(\mathrm{d}\gamma) = \mathrm{e}^{-\sigma(\Lambda)}\sum_{n=0}^{\infty}\frac{1}{n!}\int_{\Lambda^n} F(\{x_1,\dots,x_n\})\sigma(\mathrm{d}x_1)\cdots\sigma(\mathrm{d}x_n). \tag{2.12}$$

We define on the set $\mathcal{F}\Omega^n$ the L^2-scalar product with respect to the Poisson measure

$$(W^{(1)}, W^{(2)})_{L^2_{\pi_\sigma}\Omega^n} := \int_{\Gamma_X} \langle W^{(1)}(\gamma), W^{(2)}(\gamma)\rangle_{\wedge^n T_\gamma \Gamma_X}\,\pi_\sigma(\mathrm{d}\gamma). \tag{2.13}$$

As easily seen, for each $W \in \mathcal{F}\Omega^n$, there exists $\varphi \in \mathcal{D}$, $\varphi \ge 0$, such that

$$|\langle W(\gamma), W(\gamma)\rangle_{\wedge^n T_\gamma \Gamma_X}| \le \langle \varphi^{\otimes n}, \gamma^{\otimes n}\rangle.$$

Hence, the function under the sign of integral in (2.13) indeed belongs to $L^1(\Gamma_X; \pi_\sigma)$, since the Poisson measure has all moments finite. Moreover, $(W, W)_{L^2_{\pi_\sigma}\Omega^n} > 0$ if W is not identically zero. Hence, we can define the Hilbert space

$$L^2_{\pi_\sigma}\Omega^n := L^2(\Gamma_X \to \wedge^n(T\Gamma_X); \pi_\sigma)$$

as the closure of $\mathcal{F}\Omega^n$ in the norm generated by the scalar product (2.13).

We will give now an isomorphic description of the space $L^2_{\pi_\sigma}\Omega^n$ via the space $L^2_{\pi_\sigma}(\Gamma_X) := L^2(\Gamma_X \to \mathbb{R}; \pi_\sigma)$ and some special spaces of square-integrable forms on X^m, $m = 1, \dots, n$.

We need first some preparations. Let X^m be the mth Cartesian power of the manifold X. We have

$$\wedge^n(T_{(x_1,\dots,x_m)}X^m) = \bigoplus_{\substack{0\le k_1,\dots,k_m\le d\\ k_1+\cdots+k_m=n}} (T_{x_1}X)^{\wedge k_1} \wedge \cdots \wedge (T_{x_m}X)^{\wedge k_m}. \tag{2.14}$$

For an n-form ω on X^m, we denote by $\omega(x_1,\dots,x_m)_{k_1,\dots,k_m}$ the corresponding component of $\omega(x_1,\dots,x_m)$ in the decomposition (2.14).

Let

$$\tilde{X}^m := \{\bar{x} = (x_1,\dots,x_m) \in X^m | x_i \ne x_j \text{ if } i \ne j\}.$$

We introduce a set $\Psi^n_0(\tilde{X}^m)$ (resp. $\Psi^n_0(X^m)$) of bounded n-forms ω over X^m which have compact support, smooth on $\tilde{X}^m$ (resp. on X^m), and satisfy the following assumptions:

1. $\omega(x_1,\dots,x_m)_{k_1,\dots,k_m} = 0$ if $k_j = 0$ for some $j \in \{1,\dots,m\}$;

2. ω is symmetric:

$$\omega(x_1,\dots,x_m)=\omega(x_{\sigma(1)},\dots,x_{\sigma(m)})\quad \text{for each}\ \sigma\in S_m \tag{2.15}$$

(we identify the spaces $\wedge^n(T_{(x_1,\dots,x_m)}X^m)$ and $\wedge^n(T_{(x_{\sigma(1)},\dots,x_{\sigma(m)})}X^m)$ see (2.14) and the explanation just after formula (2.5)).

Evidently, $\Psi_0^n(X^m)\subset\Psi_0^n(\tilde X^m)$. Let $:\gamma^{\otimes m}:$ be the measure on X^m given by

$$:\gamma^{\otimes m}:(\mathrm{d}x_1,\dots,\mathrm{d}x_m):=\sum_{\{y_1,\dots,y_m\}\subset\gamma}\varepsilon_{y_1}\hat\otimes\cdots\hat\otimes\varepsilon_{y_m}(\mathrm{d}x_1,\dots,\mathrm{d}x_m),$$

where

$$\varepsilon_{y_1}\hat\otimes\cdots\hat\otimes\varepsilon_{y_m}(\mathrm{d}x_1,\dots,\mathrm{d}x_m):=\frac{1}{m!}\sum_{\sigma\in S_m}\varepsilon_{y_{\sigma(1)}}\otimes\cdots\otimes\varepsilon_{y_{\sigma(m)}}(\mathrm{d}x_1,\dots,\mathrm{d}x_m).$$

We will use the notation

$$\mathbb{T}^{(n)}_{\{x_1,\dots,x_m\}}X^m:=\bigoplus_{\substack{1\le k_1,\dots,k_m\le d\\ k_1+\cdots+k_m=n}}(T_{x_1}X)^{\wedge k_1}\wedge\cdots\wedge(T_{x_m}X)^{\wedge k_m}. \tag{2.16}$$

By virtue of (2.5), we have

$$\wedge^n(T_\gamma\Gamma_X)=\bigoplus_{m=1}^{n}\bigoplus_{\{x_1,\dots,x_m\}\subset\gamma}\mathbb{T}^{(n)}_{\{x_1,\dots,x_m\}}X^m. \tag{2.17}$$

For $W\in\mathcal{F}\Omega^n$, we denote by $W_m(\gamma)\in\bigoplus_{\{x_1,\dots,x_m\}\subset\gamma}\mathbb{T}^{(n)}_{\{x_1,\dots,x_m\}}X^m$ the corresponding component of $W(\gamma)$ in the decomposition (2.17). Thus, for $\{x_1,\dots,x_m\}\subset\gamma$, $W_m(\gamma)(x_1,\dots,x_m)$ is equal to the projection of $W(\gamma)\in\wedge^n(T_\gamma\Gamma_X)$ onto the subspace $\mathbb{T}^{(n)}_{\{x_1,\dots,x_m\}}X^m$. For $\bar x=(x_1,\dots,x_m)\in\tilde X^m$ we set $\{\bar x\}:=\{x_1,\dots,x_m\}$.

Lemma 2.1. *For $W,V\in\mathcal{F}\Omega^n$, we have*

$$(W(\gamma),V(\gamma))_{\wedge^n(T_\gamma\Gamma_X)}=\sum_{m=1,\dots,n}\int_{X^m}(W_m(\gamma)(\bar x),V_m(\gamma)(\bar x))_{\mathbb{T}^{(n)}_{\{\bar x\}}X^m}:\gamma^{\otimes m}:(\mathrm{d}\bar x). \tag{2.18}$$

The proof can be obtained by a direct calculation.

Let us remark that each $\omega\in\Psi_0^n(X^m)$ generates a cylinder form $W\in\mathcal{F}\Omega^n$ by the formula

$$W_k(\gamma)(x_1,\dots,x_k)=\begin{cases}\omega(x_1,\dots,x_m), & k=m,\\ 0, & k\neq m.\end{cases}$$

Let us denote by $L^2_\sigma\Psi_0^n(X^m)$ the space obtained as the completion of $\Psi_0^n(X^m)$ in the L^2-scalar product with respect to the measure $\sigma^{\otimes m}$. Evidently, $\Psi_0^n(\tilde X^m)$ is a dense subset of $L^2_\sigma\Psi_0^n(X^m)$. We have the following proposition.

Proposition 2.2. *The space $L^2_{\pi_\sigma}\Omega^n$ is unitarily isomorphic to the space*

$$L^2_{\pi_\sigma}(\Gamma_X)\otimes\left[\bigoplus_{m=1}^{n}L^2_\sigma\Psi^n(X^m)\right]=\bigoplus_{m=1}^{n}L^2_{\pi_\sigma}(\Gamma_X)\otimes L^2_\sigma\Psi^n(X^m), \tag{2.19}$$

where the corresponding isomorphism I^n is defined by the formula

$$I^n_m W(\gamma,\bar{x}):=(m!)^{-1/2}W_m(\gamma\cup\{\bar{x}\})(\bar{x}),\quad m=1,\ldots,n. \tag{2.20}$$

Here, $I^n_m W:=(I^n W)_m$ is the mth component of $I^n V$ in the decomposition (2.19).

Remark 2.5. *Actually, the formula* (2.20) *makes sense only for $\bar{x}\in\tilde{X}^m$. However, since the set $X^m\setminus\tilde{X}^m$ is of zero $\sigma^{\otimes m}$ measure, this does not lead to a contradiction.*

Proof. First, we recall an extension of the Mecke identity (e.g. [26]) to the case of functions of several variables [31]:

$$\int_{\Gamma_X}\left[\int_{X^m}f(\gamma,\bar{x}):\gamma^{\otimes m}:(\mathrm{d}\bar{x})\right]\pi_\sigma(\mathrm{d}\gamma)$$
$$=(m!)^{-1}\int_{\Gamma_X}\left[\int_{X^m}f(\gamma\cup\{\bar{x}\},\bar{x})\sigma^{\otimes m}(\mathrm{d}\bar{x})\right]\pi_\sigma(\mathrm{d}\gamma), \tag{2.21}$$

where $f:\Gamma_X\times X^m\to\mathbb{R}$ is a measurable function for which at least one of the double-integrals in (2.21) exists (this formula can be easily proved by a direct calculation using (2.12) for $f(\gamma,\bar{x})=F(\gamma)g(\bar{x})$, where $F(\gamma)$ is bounded and cylindrical and $g(\bar{x})$ is bounded and has compact support).

Next, let us specify the scalar product of two cylinder n-forms $W,V\in\mathcal{F}\Omega^n$. We have, according to (2.18),

$$(W(\gamma),V(\gamma))_{\wedge^n(T_\gamma\Gamma_X)}=\sum_{m=1}^{n}\int_{X^m}(W_m(\gamma)(\bar{x}),V_m(\gamma)(\bar{x}))_{\bar{x}}:\gamma^{\otimes m}:(\mathrm{d}\bar{x})$$
$$=\sum_{m=1}^{n}\int_{X^m}(W_m(\gamma\cup\{\bar{x}\})(\bar{x}),V_m(\gamma\cup\{\bar{x}\})(\bar{x}))_{\bar{x}}:\gamma^{\otimes m}:(\mathrm{d}\bar{x}), \tag{2.22}$$

where $(\bullet,\bullet)_{\bar{x}}:=(\bullet,\bullet)_{\mathbb{T}^{(n)}_{\{\bar{x}\}}X^m}$ (we used the evident equality $\gamma\cup\{\bar{x}\}=\gamma$ for $\{\bar{x}\}\subset\gamma$). The application of the Mecke identity (2.21) to the function

$$f(\gamma,\bar{x})=(W_m(\gamma\cup\{\bar{x}\})(\bar{x}),V_m(\gamma\cup\{\bar{x}\})(\bar{x}))_{\bar{x}}$$

shows that

$$(W,V)_{L^2_{\pi_\sigma}\Omega^n}$$
$$=\sum_{m=1}^{n}(m!)^{-1}\int_{\Gamma_X}\int_{X^m}(W_m(\gamma\cup\{\bar{x}\})(\bar{x}),V_m(\gamma\cup\{\bar{x}\})(\bar{x}))_{\bar{x}}\sigma^{\otimes m}(\mathrm{d}\bar{x})\pi_\sigma(\mathrm{d}\gamma).$$

The space $\mathcal{F}\Omega^n$ is dense in $L^2_{\pi_\sigma}\Omega^n$, and so it remains only to show that $I^n(\mathcal{F}\Omega^n)$ is a dense subspace of $\oplus_{m=1}^n L^2_{\pi_\sigma}(\Gamma_X)\otimes L^2_\sigma\Psi^n(X^m)$, i.e., $I^n_m(\mathcal{F}\Omega^n)$ is a dense subspace of $L^2_{\pi_\sigma}(\Gamma_X)\otimes L^2_\sigma\Psi^n(X^m)$, $m=1,\dots,n$.

For $F\in\mathcal{FC}$ and $\omega\in\Psi^n_0(X^m)$, we define a form W by setting

$$W_k(\gamma):=0\quad\text{for } k\neq m,\qquad W_m(\gamma)(\bar{x}):=(m!)^{1/2}F(\gamma\setminus\{\bar{x}\})\omega(\bar{x}). \tag{2.23}$$

Evidently, we have $W\in\mathcal{F}\Omega^n$ and

$$I^n_k W(\gamma,\bar{x})=0\quad\text{for } k\neq m,\qquad I^n_m W(\gamma,\bar{x})=F(\gamma)\omega(\bar{x}) \tag{2.24}$$

for each $\gamma\in\Gamma_X$ and $\bar{x}\in\tilde{X}^m$ such that $\{\bar{x}\}\cap\gamma=\emptyset$. Since γ is a set of zero σ measure, we conclude from (2.24) that

$$I^n_m W=F\otimes\omega.$$

Noting that the linear span of such $F\otimes\omega$ is dense in $L^2_{\pi_\sigma}(\Gamma_X)\otimes L^2_\sigma\Psi^n(X^m)$, we obtain the result. □

In what follows, we will denote by $\mathcal{D}\Omega^n$ the linear span of forms W defined by (2.23), $m=1,\dots,n$. As we already noticed in the proof of Proposition 2.2, $\mathcal{D}\Omega^n$ is a subset of $\mathcal{F}\Omega^n$ and is dense in $L^2_{\pi_\sigma}\Omega^n$.

Corollary 2.1. *We have the unitary isomorphism*

$$\mathcal{I}^n : L^2_{\pi_\sigma}\Omega^n\to \operatorname{Exp}(L^2(X;\sigma))\otimes\left[\bigoplus_{m=1}^n L^2_\sigma\Psi^n(X^m)\right]$$

given by

$$\mathcal{I}^n:=(U\otimes\mathbf{1})I^n,$$

where U is the Wiener–Itô–Segal isomorphism between the Poisson space $L^2_{\pi_\sigma}(\Gamma_X)$ and the symmetric Fock space $\operatorname{Exp}(L^2(X;\sigma))$ *over* $L^2(X;\sigma)$ (*see, e.g.,* [8]).

3. Dirichlet operators on differential forms over configuration spaces

In this section, we introduce Dirichlet operators associated with the Poisson measure on Γ_X which act in the spaces of square-integrable forms. These operators generalize the notions of Bochner and de Rham–Witten Laplacians on finite-dimensional manifolds. But first, we recall some known facts and definitions concerning the usual Dirichlet operator of the Poisson measure and Laplace operators on differential forms over finite-dimensional manifolds.

3.1. The intrinsic Dirichlet operator on functions

In this section, we recall some theorems from [8] which concern the intrinsic Dirichlet operator in the space $L^2_{\pi_\sigma}(\Gamma_X)$, to be used later.

Let us recall that the logarithmic derivative of the measure σ is given by the vector field

$$X \ni x \mapsto \beta_\sigma(x) := \frac{\nabla^X \rho(x)}{\rho(x)} \in T_x X$$

(where as usual $\beta_\sigma := 0$ on $\{\rho = 0\}$). We wish now to define a logarithmic derivative of the Poisson measure, and for this we need a generalization of the notion of vector field.

For each $\gamma \in \Gamma_X$, consider the triple

$$T_{\gamma,\infty}\Gamma_X \supset T_\gamma \Gamma_X \supset T_{\gamma,0}\Gamma_X.$$

Here, $T_{\gamma,0}\Gamma_X$ consists of all finite sequences from $T_\gamma \Gamma_X$, and $T_{\gamma,\infty}\Gamma_X := (T_{\gamma,0}\Gamma_X)'$ is the dual space, which consists of all sequences $V(\gamma) = (V(\gamma)_x)_{x\in\gamma}$, where $V(\gamma)_x \in T_x X$. The pairing between any $V(\gamma) \in T_{\gamma,\infty}\Gamma_X$ and $v(\gamma) \in T_{\gamma,0}\Gamma_X$ with respect to the zero space $T_\gamma \Gamma_x$ is given by

$$\langle V(\gamma), v(\gamma)\rangle_\gamma = \sum_{x\in\gamma} \langle V(\gamma)_x, v(\gamma)_x\rangle_x$$

(the series is, in fact, finite). From now on, under a vector field over Γ_X we will understand mappings of the form $\Gamma_X \ni \gamma \mapsto V(\gamma) \in T_{\gamma,\infty}\Gamma_X$.

The logarithmic derivative of the Poisson measure π_σ is defined as the vector field

$$\Gamma_X \ni \gamma \mapsto B_{\pi_\sigma}(\gamma) = (\beta_\sigma(x))_{x\in\gamma} \in T_{\gamma,\infty}\Gamma_X \tag{3.1}$$

(i.e., the logarithmic derivative of the Poisson measure is the lifting of the logarithmic derivative of the underlying measure).

The following theorem is a version of Theorem 3.1 in [8] (for more general classes of functions and vector fields).

Theorem 3.1 (Integration by parts formula on the Poisson space). *For arbitrary $F^{(1)}, F^{(2)} \in \mathcal{F}\mathcal{C}$ and a smooth cylinder vector field $V \in \mathcal{F}\mathcal{V}(:= \mathcal{F}\Omega^1)$, we have*

$$\int_{\Gamma_X} \nabla_V^\Gamma F^{(1)}(\gamma) F^{(2)}(\gamma)\pi_\sigma(\mathrm{d}\gamma) = -\int_{\Gamma_X} F^{(1)}(\gamma)\nabla_V^\Gamma F^{(2)}(\gamma)\pi_\sigma(\mathrm{d}\gamma)$$
$$-\int_{\Gamma_X} F^{(1)}(\gamma)F^{(2)}(\gamma)[\langle B_{\pi_\sigma}(\gamma), V(\gamma)\rangle_\gamma + \mathrm{div}^\Gamma V(\gamma)]\pi_\sigma(\mathrm{d}\gamma),$$

where the divergence $\mathrm{div}^\Gamma V(\gamma)$ *of the vector field V is given by*

$$\mathrm{div}\, V(\gamma) = \sum_{x\in\gamma} \mathrm{div}_x^X V(\gamma) = \langle \mathrm{div}_\bullet^X V(\gamma), \gamma\rangle, \qquad \mathrm{div}_x^X V(\gamma) := \mathrm{div}^X V_x(\gamma, x), \quad x \in \gamma$$

with div^X *denoting the divergence on X with respect to the volume measure m.*

Proof. The theorem follows from formula (2.12) and the usual integration by parts formula on the space $L^2(\Lambda^n, \sigma^{\otimes n})$ (see also the proof of Theorem 3.3). □

Following [8], we consider the intrinsic pre-Dirichlet form on the Poisson space

$$\mathcal{E}_{\pi_\sigma}(F^{(1)}, F^{(2)}) = \int_{\Gamma_X} \langle \nabla^\Gamma F^{(1)}(\gamma), \nabla^\Gamma F^{(2)}(\gamma)\rangle_\gamma \pi_\sigma(\mathrm{d}\gamma) \tag{3.2}$$

with domain $D(\mathcal{E}_{\pi_\sigma}) := \mathcal{FC}$. By using the fact that the measure π_σ has all moments finite and noting that there exists a function $\varphi \in \mathcal{D}$, $\varphi \geq 0$, such that

$$|\langle \nabla^\Gamma F^{(1)}(\gamma), \nabla^\Gamma F^{(2)}(\gamma)\rangle_\gamma| \leq \langle \varphi, \gamma\rangle,$$

one concludes that the expression (3.2) is well defined.

Let H_σ denote the Dirichlet operator in the space $L^2(X;\sigma)$ associated to the pre-Dirichlet form

$$\mathcal{E}_\sigma(\varphi, \psi) = \int_X \langle \nabla^X \varphi(x), \nabla^X \psi(x)\rangle_x \sigma(\mathrm{d}x), \quad \varphi, \psi \in \mathcal{D}.$$

This operator acts as follows:

$$H_\sigma \varphi(x) = -\Delta^X \varphi(x) - \langle \beta_\sigma(x), \nabla^X \varphi(x)\rangle_x, \quad \varphi \in \mathcal{D},$$

where $\Delta^X := \mathrm{div}^X \nabla^X$ is the Laplace–Beltrami operator on X.

Then, by using Theorem 3.1, one gets

$$\mathcal{E}_{\pi_\sigma}(F^{(1)}, F^{(2)}) = \int_{\Gamma_X} H_{\pi_\sigma} F^{(1)}(\gamma) F^{(2)}(\gamma) \pi_\sigma(\mathrm{d}\gamma), \quad F^{(1)}, F^{(2)} \in \mathcal{FC}. \tag{3.3}$$

Here, the intrinsic Dirichlet operator H_{π_σ} is given by

$$\begin{aligned} H_{\pi_\sigma} F(\gamma) &:= \sum_{x\in\gamma} H_{\sigma,x} F(\gamma) \equiv \langle H_{\sigma,\bullet} F(\gamma), \gamma\rangle, \\ H_{\sigma,x} F(\gamma) &:= H_\sigma F_x(\gamma, x), \quad x \in \gamma, \end{aligned} \tag{3.4}$$

so that the operator H_{π_σ} is the lifting to $L^2_{\pi_\sigma}(\Gamma_X)$ of the operator H_σ in $L^2(X;\sigma)$.

Upon (3.3), the pre-Dirichlet form $\mathcal{E}_{\pi_\sigma}$ is closable, and we preserve the notation for the closure of this form.

Theorem 3.2 (Albeverio, Kondrative and Röckner [8]). *Suppose that* $(H_\sigma, \mathcal{D})$ *is essentially self-adjoint on* $L^2(X;\sigma)$. *Then, the operator* H_{π_σ} *is essentially self-adjoint on* $\mathcal{FC}$.

Remark 3.1. *This theorem was proved in* [8, *Theorem* 5.3]. (*We have already mentioned in Remark* 2.1 *that the inclusion* $\mathcal{FC}_\mathrm{b}^\infty(\mathcal{D}, \Gamma_X) \subset \mathcal{FC}$ *holds*.) *We would like to stress that this result is based upon the theorem which says that the image of the operator* H_{π_σ} *under the isomorphism U between the Poisson space and the Fock space* $\mathrm{Exp}(L^2(X;\sigma))$ *is the differential second quantization* $\mathrm{d\,Exp}\, H_\sigma$ *of the operator* H_σ.

Remark 3.2. *The condition of Theorem* 3.2 *is satisfied if, e.g.,*

$$\|\beta_\sigma\|_{TX} \in L^p_{\mathrm{loc}}(X;\sigma) \tag{3.5}$$

for some $p > \dim X$ (*see* [8]).

In what follows, we will suppose for simplicity that

$$\text{the function } \rho \text{ is infinitely differentiable on } X \text{ and } \rho(x) > 0 \text{ for all } x \in X. \tag{3.6}$$

Evidently, estimate (3.5) is implied by (3.6).

Finally, we mention the important fact [8] that the diffusion process which is properly associated with the Dirichlet form $(\mathcal{E}_{\pi_\sigma}, D(\mathcal{E}_{\pi_\sigma}))$ is the usual independent infinite particle process (or distorted Brownian motion on Γ_X), introduced by Doob [19].

3.2. *Laplacians on differential forms over finite-dimensional manifolds*

We recall now some facts on the Bochner and de Rham–Witten Laplacians on differential forms over a finite-dimensional manifold.

Let M be a Riemannian manifold equipped with the measure $\mu(\mathrm{d}x) = \mathrm{e}^{\phi(x)}\,\mathrm{d}x$, $\mathrm{d}x$ being the volume measure and ϕ a C^2-function on M. We consider a Hilbert bundle

$$\mathcal{H}_x \mapsto x \in M$$

over M equipped with a smooth connection, and denote by ∇ the corresponding covariant derivative in the spaces of sections of this bundle. Let $L^2(M \to \mathcal{H}; \mu)$ be the space of μ-square integrable sections. The operator

$$H_\mu^{\mathrm{B}} := \nabla_\mu^* \nabla$$

in $L^2(M \to \mathcal{H}; \mu)$, where ∇_μ^* is the adjoint of ∇, will be called the Bochner Laplacian associated with the measure μ. One can easily write the corresponding differential expression on the space of twice differentiable sections. In the case where $\phi \equiv 0$ and $\mathcal{H}_x = \wedge^n(T_xM)$, we obtain the classical Bochner Laplacian on differential forms (see, e.g., [17]).

Now, let d be the exterior differential in spaces of differential forms over M. The operator

$$H_\mu^{\mathrm{R}} := d_\mu^* d + d d_\mu^*$$

acting in the space of μ-square integrable forms, where d_μ^* is the adjoint of d, will be called the de Rham Laplacian associated with the measure μ (or the Witten Laplacian associated with ϕ; see, e.g., [17]).

We will use sometimes more extended notations $H_{\mu,n}^{\mathrm{B}}(M)$, $H_{\mu,n}^{\mathrm{R}}(M)$ for the Bochner and de Rham–Witten Laplacians on the space of μ-square integrable n-forms over M.

The relation of the Bochner and de Rham–Witten Laplacians on differential forms is given by the Weitzenböck formula (cf. [17,20]), which will be recalled now.

Fix $x \in M$ and let $(e_j)_{j=1}^{\dim M}$ be an orthonormal basis in T_xM. Denote by

$$a_j : \wedge^{n+1}(T_xM) \to \wedge^n(T_xM), \qquad a_j^* : \wedge^n(T_xM) \to \wedge^{n+1}(T_xM), \tag{3.7}$$

the annihilation and creation operators, respectively, defined by

$$\begin{aligned} a_j u^{(n+1)} &= (n+1)^{1/2} \langle e_j, u^{(n+1)} \rangle_x, \quad u^{(n+1)} \in \wedge^{n+1}(T_xM), \\ a_j^* u^{(n)} &= (n+1)^{1/2} e_j \wedge u^{(n)}, \quad u^{(n)} \in \wedge^n(T_xM). \end{aligned} \tag{3.8}$$

The pairing in the expression $\langle e_j, u^{(n+1)}\rangle_x$ is carried out in the first "variable", so that a_j^* becomes the adjoint of a_j.

Let us introduce the operator $R_n(x)$ in $\wedge^n(T_xM)$ by

$$R_n(x) := \sum_{i,j,k,l=1}^{\dim M} R_{ijkl}(x) a_i^* a_j a_k^* a_l,$$

where R_{ijkl} is the curvature tensor on M. It can be shown that the definition of this operator is independent of the specific choice of basis.

Next, let $(\nabla^M \beta_\mu(x))^{\wedge n}$ be the operator in $\wedge^n(T_xM)$ given by

$$(\nabla^M \beta_\mu(x))^{\wedge n} := \nabla^M \beta_\mu(x) \otimes \mathbf{1} \cdots \otimes \mathbf{1} + \mathbf{1} \otimes \nabla^M \beta_\mu(x) \otimes \mathbf{1} \otimes \cdots \otimes \mathbf{1} + \cdots + \mathbf{1} \otimes \cdots \otimes \mathbf{1} \otimes \nabla^M \beta_\mu(x), \tag{3.9}$$

$\nabla^M \beta_\mu(x)$ being understood as an operator in T_xM.

Then, the Weitzenböck formula writes as follows:

$$H_\mu^{\mathrm{R}} \omega_n(x) = H_\mu^{\mathrm{B}} \omega_n(x) + R_\mu(x)\omega_n(x), \tag{3.10}$$

where ω_n is an n-form on X, and $R_\mu(x)\omega_n(x) = R_{\mu,n}(x)\omega_n(x)$,

$$R_{\mu,n}(x) := R_n(x) - (\nabla^M \beta_\mu(x))^{\wedge n}. \tag{3.11}$$

Remark 3.3. *The classical Weitzenböck formula is related, in fact, to the case where* $\phi \equiv 0$ (*see, e.g.,* [17,20]). *Formula* (3.10) *can be obtained by a direct calculation using similar arguments, cf.* [1].

3.3. Bochner Laplacian on forms over the configuration space

Let us consider the pre-Dirichlet form

$$\mathcal{E}_{\pi_\sigma}^{\mathrm{B}}(W^{(1)}, W^{(2)}) = \int_{\Gamma_X} \langle \nabla^\Gamma W^{(1)}(\gamma), \nabla^\Gamma W^{(2)}(\gamma)\rangle_{T_\gamma \Gamma_X \otimes \wedge^n(T_\gamma \Gamma_X)} \pi_\sigma(\mathrm{d}\gamma), \tag{3.12}$$

where $W^{(1)}, W^{(2)} \in \mathcal{F}\Omega^n$. As easily seen, there exists $\varphi \in \mathcal{D}$, $\varphi \geq 0$, such that

$$|\langle \nabla^\Gamma W^{(1)}(\gamma), \nabla^\Gamma W^{(2)}(\gamma)\rangle_{T_\gamma \Gamma_X \otimes \wedge^n(T_\gamma \Gamma_X)}| \leq \langle \varphi^{\otimes(n+1)}, \gamma^{\otimes(n+1)}\rangle,$$

so that the function under the sign of integral in (3.12) is integrable with respect to π_σ.

Theorem 3.3. *For any* $W^{(1)}, W^{(2)} \in \mathcal{F}\Omega^n$, *we have*

$$\mathcal{E}_{\pi_\sigma}^{\mathrm{B}}(W^{(1)}, W^{(2)}) = \int_{\Gamma_X} \langle H_{\pi_\sigma}^{\mathrm{B}} W^{(1)}(\gamma), W^{(2)}(\gamma)\rangle_{\wedge^n(T_\gamma \Gamma_X)} \pi_\sigma(\mathrm{d}\gamma),$$

where $H_{\pi_\sigma}^{\mathrm{B}}$ *is the operator in the space* $L^2_{\pi_\sigma}\Omega^n$ *with domain* $\mathcal{F}\Omega^n$ *given by*

$$H_{\pi_\sigma}^{\mathrm{B}} W(\gamma) = -\Delta^\Gamma W(\gamma) - \langle \nabla^\Gamma W(\gamma), B_{\pi_\sigma}(\gamma)\rangle_\gamma, \quad W \in \mathcal{F}\Omega^n. \tag{3.13}$$

Here,

$$\Delta^{\Gamma} W(\gamma) := \sum_{x\in\gamma} \Delta_x^X W(\gamma) \equiv \langle \Delta_\bullet^{\Gamma} W(\gamma), \gamma\rangle, \tag{3.14}$$

where Δ_x^X *is the Bochner Laplacian of the bundle* $\wedge^n(T_{\gamma_y}\Gamma_X) \mapsto y \in \mathcal{O}_{\gamma,x}$ *with the volume measure*.

Proof. First, we note that, for $W \in \mathcal{F}\Omega^n$,

$$\Delta_x^X W(\gamma)_{[x_1,\dots,x_n]_d} := \Delta^X W_x(\gamma, x)_{[x_1,\dots,x_n]_d}, \quad x \in \gamma, \ \ [x_1, \dots, x_n]_d \subset \gamma.$$

Fix now $W^{(1)}, W^{(2)} \in \mathcal{F}\Omega^n$ and let Λ_1, Λ_2 be compact subsets of X as in Definition 2.3 corresponding to $W^{(1)}, W^{(2)}$, respectively. Let Λ be an open set in X with compact closure such that both Λ_1 and Λ_2 are subsets of Λ. Then, by using (2.12),

$$\begin{aligned}
&\int_{\Gamma_X} \langle \nabla^{\Gamma} W^{(1)}(\gamma), \nabla^{\Gamma} W^{(2)}(\gamma)\rangle_{T_\gamma\Gamma_X\otimes\wedge^n(T_\gamma\Gamma_X)}\pi_\sigma(\mathrm{d}\gamma)\\
&\quad = \mathrm{e}^{-\sigma(\Lambda)}\sum_{k=0}^{\infty}\frac{1}{k!}\int_{\Lambda^k}\sum_{i=1}^{k}\langle\nabla_{x_i}^X W^{(1)}(\{x_1,\dots,x_k\}),\\
&\qquad \nabla_{x_i}^X W^{(2)}(\{x_1,\dots,x_k\})\rangle_{T_{x_i}X\otimes\wedge^n(T_{x_1}X\oplus\cdots\oplus T_{x_k}X)}\sigma(\mathrm{d}x_1)\cdots\sigma(\mathrm{d}x_k)\\
&\quad = \mathrm{e}^{-\sigma(\Lambda)}\sum_{k=0}^{\infty}\frac{1}{k!}\sum_{i=1}^{k}\int_{\Lambda^k}\sum_{[y_1,\dots,y_n]_d\subset\{x_1,\dots,x_k\}}\langle\nabla_{x_i}^X W^{(1)}(\{x_1,\dots,x_k\})_{[y_1,\dots,y_n]_d},\\
&\qquad \nabla_{x_i}^X W^{(2)}(\{x_1,\dots,x_k\})_{[y_1,\dots,y_n]_d}\rangle_{T_{x_i}X\otimes(T_{y_1}X\wedge\cdots\wedge T_{y_n}X)}\sigma(\mathrm{d}x_1)\cdots\sigma(\mathrm{d}x_k)\\
&\quad = \mathrm{e}^{-\sigma(\Lambda)}\sum_{k=0}^{\infty}\frac{1}{k!}\sum_{i=1}^{k}\int_{\Lambda^k}\sum_{[y_1,\dots,y_n]_d\subset\{x_1,\dots,x_k\}}\langle\Delta_{x_i}^X W^{(1)}(\{x_1,\dots,x_k\})_{[y_1,\dots,y_n]_d}\\
&\qquad +\langle\nabla_{x_i}^X W^{(1)}(\{x_1,\dots,x_k\})_{[y_1,\dots,y_n]_d}, \beta_\sigma(x_i)\rangle_{x_i},\\
&\qquad W^{(2)}(\{x_1,\dots,x_k\})_{[y_1,\dots,y_n]_d}\rangle_{T_{y_1}X\wedge\cdots\wedge T_{y_n}X}\sigma(\mathrm{d}x_1)\cdots\sigma(\mathrm{d}x_k)\\
&\quad = \int_{\Gamma_X}\langle H_{\pi_\sigma}^{\mathrm{B}} W^{(1)}(\gamma), W^{(2)}(\gamma)\rangle_{\wedge^n(T_\gamma\Gamma_X)}\pi_\sigma(\mathrm{d}\gamma).
\end{aligned}$$

□

Remark 3.4. *We can rewrite the action of the operator* $H_{\pi_\sigma}^{\mathrm{B}}$ *in the two following forms*:

1. *We have from* (3.13) *and* (3.14) *that*

$$H_{\pi_\sigma}^{\mathrm{B}} W(\gamma) = \sum_{x\in\gamma} H_{\sigma,x}^{\mathrm{B}} W(\gamma) \equiv \langle H_{\sigma,\bullet}^{\mathrm{B}} W(\gamma), \gamma\rangle, \quad W(\gamma) \in \mathcal{F}\Omega^n, \tag{3.15}$$

where

$$H_{\sigma,x}^{\mathrm{B}} W(\gamma) := -\Delta_x^X W(\gamma) - \langle \nabla_x^X W(\gamma), \beta_\sigma(x)\rangle_x. \tag{3.16}$$

Thus, the operator $H_{\pi_\sigma}^{\mathrm{B}}$ *is the lifting of the Bochner Laplacian on X with the measure* σ.

2. *As easily seen, the operator $H^{\mathrm{B}}_{\pi_\sigma}$ preserves the space $\mathcal{F}\Omega^n$, and we can always take $\Lambda(H^{\mathrm{B}}_{\pi_\sigma}W)=\Lambda(W)$. Then, for any open bounded $\Lambda\supset\Lambda(W)$ (cf. Remark 2.2), we have*

$$(H^{\mathrm{B}}_{\pi_\sigma}W)_{\Lambda,\gamma}=H^{\mathrm{B}}_{\sigma^{\otimes|\Lambda\cap\gamma|}}(X^{|\Lambda\cap\gamma|})W_{\Lambda,\gamma}, \tag{3.17}$$

where $H^{\mathrm{B}}_{\sigma^{\otimes|\Lambda\cap\gamma|}}(X^{|\Lambda\cap\gamma|})$ is the Bochner Laplacian of the manifold $X^{|\Lambda\cap\gamma|}$ with the product measure $\sigma^{\otimes|\Lambda\cap\gamma|}$ (cf. (2.10)). The equality (3.17) holds on $\mathcal{O}_{\gamma,x_1}\times\cdots\times\mathcal{O}_{\gamma,x_{|\Lambda\cap\gamma|}}$, where $\{x_1,\dots,x_{|\Lambda\cap\gamma|}\}=\Lambda\cap\gamma$. Notice that, since the operator $H^{\mathrm{B}}_{\sigma^{\otimes|\Lambda\cap\gamma|}}(X^{|\Lambda\cap\gamma|})$ acts locally on (smooth) forms on $X^{|\Lambda\cap\gamma|}$, the expression on the right-hand side of (3.17) is well defined as a form on $\mathcal{O}_{\gamma,x_1}\times\cdots\times\mathcal{O}_{\gamma,x_{|\Lambda\cap\gamma|}}$.

It follows from Theorem 3.3 that the pre-Dirichlet form $\mathcal{E}^{\mathrm{B}}_{\pi_\sigma}$ is closable in the space $L^2_{\pi_\sigma}\Omega^n$. The generator of its closure (being actually the Friedrichs extension of the operator $H^{\mathrm{B}}_{\pi_\sigma}$, for which we will use the same notation) will be called the Bochner Laplacian on n-forms over Γ_X corresponding to the Poisson measure π_σ.

For linear operators A and B acting in Hilbert spaces $\mathcal{H}$ and $\mathcal{K}$, respectively, we introduce the operator $A\boxplus B$ in $\mathcal{H}\otimes\mathcal{K}$ by

$$A\boxplus B:=A\otimes\mathbf{1}+\mathbf{1}\otimes B,\qquad \mathrm{Dom}(A\boxplus B):=\mathrm{Dom}(A)\otimes_{\mathrm{a}}\mathrm{Dom}(B),$$

where $\otimes_{\mathrm{a}}$ stands for the algebraic tensor product. Next, for operators $A_1,\dots,A_n$ acting in Hilbert spaces $\mathcal{H}_1,\dots,\mathcal{H}_n$, respectively, let $\oplus_{i=1}^n A_i$ denote the operator in $\oplus_{i=1}^n H_i$ given by

$$\left(\bigoplus_{i=1}^n A_i\right)(f_1,\dots,f_n)=(A_1f_1,\dots,A_nf_n),\qquad f_i\in\mathrm{Dom}(A_i).$$

Theorem 3.4.

1. *On $\mathcal{D}\Omega^n$ we have*

$$H^{\mathrm{B}}_{\pi_\sigma}=(I^n)^{-1}\left[H_{\pi_\sigma}\boxplus\left(\bigoplus_{m=1}^n H^{\mathrm{B}}_{\sigma,(n,m)}\right)\right]I^n, \tag{3.18}$$

where $H^{\mathrm{B}}_{\sigma,(n,m)}$ denotes the restriction of the Bochner Laplacian $H^{\mathrm{B}}_{\sigma^{\otimes m},n}(X^m)$ acting in the space $L^2(X^m\to\wedge^n(TX^m);\sigma^{\otimes m})$ to the subspace $L^2_\sigma\Psi^n(X^m)$.

2. *Suppose that, for each $m=1,\dots,n$, the Bochner Laplacian $H^{\mathrm{B}}_{\sigma,m}(X)$ is essentially self-adjoint on the set of smooth forms with compact support. Then, $\mathcal{D}\Omega^n$ is a domain of essential self-adjointness of $H^{\mathrm{B}}_{\pi_\sigma}$, and the equality (3.18) holds for the closed operators $H^{\mathrm{B}}_{\pi_\sigma}$ and $H_{\pi_\sigma}\boxplus(\oplus_{m=1}^n H^{\mathrm{B}}_{\sigma,(n,m)})$, where the latter operator is closed from its domain of essential self-adjointness $I^n(\mathcal{D}\Omega^n)$.*

Remark 3.5. *The essential self-adjointness of the Bochner Laplacian H^{B}_σ on the set of smooth forms with compact support is well known in the case where σ is the volume measure (see, e.g., [20]). More generally, it is sufficient to assume that β_σ, together with its derivatives up to order 2, is bounded.*

Proof of Theorem 3.4.

1. Let $W \in \mathcal{D}\Omega^n$ be given by the formula (2.23). Then, using (3.15), (3.16) and (3.4), we get

$$
\begin{aligned}
&(H^{\mathrm{B}}_{\pi_\sigma} W)_k(\gamma) = 0 \quad \text{for } k \neq m, \qquad (H^{\mathrm{B}}_{\pi_\sigma} W)_m(\gamma)(\bar{x}) = \left(\sum_{x\in\gamma} H^{\mathrm{B}}_{\sigma,x} W \right)_m (\gamma)(\bar{x}) \\
&\quad = \left(\sum_{x\in\gamma\setminus\{\bar{x}\}} H^{\mathrm{B}}_{\sigma,x} W \right)_m (\gamma)(\bar{x}) + \left(\sum_{x\in\{\bar{x}\}} H^{\mathrm{B}}_{\sigma,x} W \right)_m (\gamma)(\bar{x}) \\
&\quad = (m!)^{1/2} \left(\sum_{x\in\gamma\setminus\{\bar{x}\}} H_{\sigma,x} F \right) (\gamma\setminus\{\bar{x}\})\omega(\bar{x}) + (m!)^{1/2} F(\gamma\setminus\{\bar{x}\}) \\
&\qquad \times \left(\sum_{x\in\{\bar{x}\}} H^{\mathrm{B}}_{\sigma,x} \omega \right)(\bar{x}) = (m!)^{1/2} (H_{\pi_\sigma} F)(\gamma\setminus\{\bar{x}\})\omega(\bar{x}) \\
&\qquad\qquad + (m!)^{1/2} F(\gamma\setminus\{\bar{x}\})(H^{\mathrm{B}}_{\sigma,(n,m)}\omega)(\bar{x}).
\end{aligned}
\tag{3.19}
$$

Notice that the Bochner Laplacian in the space $L^2(X^m \to \wedge^n(TX^m); \sigma^{\otimes m})$ leaves the set $\Psi^n_0(X^m)$ invariant. Therefore,

$$
(I^n_k H^{\mathrm{B}}_{\pi_\sigma} W)(\gamma,\bar{x}) = \begin{cases} 0 & \text{for } k \neq m, \\ (H_{\pi_\sigma} F)(\gamma)\omega(\bar{x}) + F(\gamma)(H^{\mathrm{B}}_{\sigma,(n,m)}\omega)(\bar{x}) & \text{for } k = m. \end{cases}
\tag{3.20}
$$

Hence, by virtue of (2.24), we get

$$
\left(\left[H_{\pi_\sigma} \boxplus \left(\bigoplus_{i=1}^{n} H^{\mathrm{B}}_{\sigma,(n,i)} \right) \right] I^n W \right)_k (\gamma,\bar{x}) = (I^n_k H^{\mathrm{B}}_{\pi_\sigma} W)(\gamma,\bar{x}), \quad k = 1,\dots,n,
$$

which proves (3.18).

2. Let $\Omega^n_0(X^m)$ denote the set of all smooth forms $\omega : X^m \to \wedge^n(X^m)$ with compact support. It is not hard to see that the essential self-adjointness of $H^{\mathrm{B}}_{\sigma,m}(X)$ for each $m = 1,\dots,n$ implies that

$$
\text{the Bochner Laplacian } H^{\mathrm{B}} := H^{\mathrm{B}}_{\sigma^{\otimes m},n}(X^m) \text{ is essentially self-adjoint on } \Omega^n_0(X^m).
\tag{3.21}
$$

Indeed, by using the decomposition (2.14), we have

$$
\begin{aligned}
&L^2(X^m \to \wedge^n(TX^m); \sigma^{\otimes m}) \\
&\quad = L^2(X^m \ni (x_1,\dots,x_m) \to \wedge^n(T_{(x_1,\dots,x_m)}X^m); \sigma^{\otimes m}) \\
&\quad = \bigoplus_{\substack{0\le k_1,\dots,k_m\le d \\ k_1+\cdots+k_m = n}} L^2(X^m \ni (x_1,\dots,x_m) \to (T_{x_1}X)^{\wedge k_1} \wedge \cdots \wedge (T_{x_m}X)^{\wedge k_m}; \sigma^{\otimes m}),
\end{aligned}
$$

and it is enough to show that the Bochner Laplacian H^{B} is essentially self-adjoint in each space

$$L^2(X^m \ni (x_1, \dots, x_m) \to (T_{x_1}X)^{\wedge k_1} \wedge \cdots \wedge (T_{x_m}X)^{\wedge k_m}; \sigma^{\otimes m}) \tag{3.22}$$

on the set of smooth forms.

On the other hand, by using the essential self-adjointness of each operator $H^{\mathrm{B}}_{\sigma,m}(X)$ on $\Omega_0^m(X)$ and that of the operator H_σ in the space $L^2(X;\sigma)$ on the set $\mathcal{D}$ (Remark 3.2), we conclude from the theory of operators admitting separation of variables [15, Chapter 6] that the operator

$$H^{\mathrm{B}}_{\sigma,k_1}(X) \boxplus \cdots \boxplus H^{\mathrm{B}}_{\sigma,k_m}(X), \qquad H^{\mathrm{B}}_{\sigma,0}(X) := H_\sigma \tag{3.23}$$

is essentially self-adjoint in the space

$$\begin{aligned} &L^2(X \to \wedge^{k_1}(TX);\sigma) \otimes \cdots \otimes L^2(X \to \wedge^{k_m}(TX);\sigma) \\ &\quad = L^2(X^m \ni (x_1,\dots,x_m) \to (T_{x_1}X)^{\wedge k_1} \otimes \cdots \otimes (T_{x_m}X)^{\wedge k_m}; \sigma^{\otimes m}) \end{aligned} \tag{3.24}$$

on the algebraic product of the domains of the operators $H^{\mathrm{B}}_{\sigma,k_i}(X)$.

Next, we note that, for each $(x_1,\dots,x_m) \in \tilde{X}^m$, there exists an intrinsic unitary isomorphism

$$\mathrm{Iso}_{k_1,\dots,k_m} : (T_{x_1}X)^{\wedge k_1} \otimes \cdots \otimes (T_{x_m}X)^{\wedge k_m} \to (T_{x_1}X)^{\wedge k_1} \wedge \cdots \wedge (T_{x_m}X)^{\wedge k_m}$$

that is given by the formula

$$\begin{aligned} &\mathrm{Iso}_{k_1,\dots,k_m}(u_1^{(1)} \wedge \cdots \wedge u_{k_1}^{(1)}) \otimes \cdots \otimes (u_1^{(m)} \wedge \cdots \wedge u_{k_m}^{(m)}) \\ &\quad := \sqrt{\frac{(k_1+\cdots+k_m)!}{k_1!\cdots k_m!}}\, u_1^{(1)} \wedge \cdots \wedge u_{k_1}^{(1)} \wedge \cdots \wedge u_1^{(m)} \wedge \cdots \wedge u_{k_m}^{(m)}, \quad u_j^{(i)} \in T_{x_i}X, \end{aligned}$$

and then it is extended by linearity. As easily seen, this definition is independent of the representation of a vector from $(T_{x_1}X)^{\wedge k_1} \otimes \cdots \otimes (T_{x_m}X)^{\wedge k_m}$. Hence, for any $(k_1,\dots,k_m)$, we can construct the unitary $\mathcal{U}_{k_1,\dots,k_m}$ between the spaces (3.24) and (3.22) by setting

$$(\mathcal{U}_{k_1,\dots,k_m}F)(x_1,\dots,x_m) := \mathrm{Iso}_{k_1,\dots,k_m}(F(x_1,\dots,x_m)).$$

Under this unitary, the operator (3.23) goes over into the operator H^{B} in the space (3.22), while the image of its domain consists of linear combinations of the form $\mathcal{U}_{k_1,\dots,k_m}(\omega^{(k_1)} \otimes \cdots \otimes \omega^{(k_m)})$, $\omega^{(k_i)} \in \Omega_0^{k_i}(X)$. From here, the assertion (3.21) follows.

Let $\hat{L}^2(X^m \to \wedge^n(TX^m);\sigma^{\otimes m})$ denote the subspace of $L^2(X^m \to \wedge^n(TX^m);\sigma^{\otimes m})$ consisting of all symmetric forms, i.e., the forms $\omega \in L^2(X^m \to \wedge^n(TX^m);\sigma^{\otimes m})$ for which the equality (2.15) holds for $\sigma^{\otimes m}$-a.a. $(x_1,\dots,x_m) \in X^m$. Evidently, the orthogonal projection P_m^n onto this subspace is given by the formula

$$(P_m^n\omega)(x_1,\dots,x_m) = \frac{1}{m!}\sum_{\sigma \in S_m} \omega(x_{\sigma(1)},\dots,x_{\sigma(m)}), \tag{3.25}$$

and

$$P_m^n \Omega_0^n(X^m) = \Omega_{0,\mathrm{sym}}^n(X^m), \tag{3.26}$$

where $\Omega_{0,\mathrm{sym}}^n(X^m)$ denotes the set of symmetric smooth forms $\omega : X^m \to \wedge^n(X^m)$ with compact support.

The assertion (3.21) and the nonnegative definiteness of H^{B} yield that the set $(H^{\mathrm{B}} + \mathbf{1})\Omega_0^n(X^m)$ is dense in $L^2(X^m \to \wedge^n(TX^m); \sigma^{\otimes m})$ (see, e.g., [32, Section 10.1]). Therefore, the set $P_m^n(H^{\mathrm{B}} + \mathbf{1})\Omega_0^n(X^m)$ is dense in $\hat{L}^2(X^m \to \wedge^n(TX^m); \sigma^{\otimes m})$. But upon (3.25) and (3.26),

$$P_m^n(H^{\mathrm{B}} + \mathbf{1})\Omega_0^n(X^m) = (H^{\mathrm{B}} P_m^n + P_m^n)\Omega_0^n(X^m) = (H^{\mathrm{B}} + \mathbf{1})\Omega_{0,\mathrm{sym}}^n(X^m),$$

which implies that the Bochner Laplacian H^{B} in the space $\hat{L}^2(X^m \to \wedge^n(TX^m); \sigma^{\otimes m})$ is essentially self-adjoint on $\Omega_{0,\mathrm{sym}}^n(X^m)$.

Because H^{B} acts invariantly on the subspace $L_\sigma^2 \Psi^n(X^m)$ and also on its orthogonal complement in $\hat{L}^2(X^m \to \wedge^n(TX^m); \sigma^{\otimes m})$, we conclude that $H_{\sigma,(n,m)}^{\mathrm{B}}$ is essentially self-adjoint on $\Psi_0^n(X^m)$. Consequently, the operator $\oplus_{m=1}^n H_{\sigma,(n,m)}^{\mathrm{B}}$ is essentially self-adjoint on the direct sum of the sets $\Psi_0^n(X^m)$, $m = 1, \ldots, n$.

Finally, taking to notice that the operator H_{π_σ} is essentially self-adjoint on $\mathcal{FC}$ (Theorem 3.2), we conclude again from the theory of operators admitting separation of variables that $I^n(\mathcal{D}\Omega^n)$ is a domain of essential self-adjointness of the operator $H_{\pi_\sigma} \boxplus (\oplus_{m=1}^n H_{\sigma,(n,m)}^{\mathrm{B}})$ in the space $L_{\pi_\sigma}^2(\Gamma_X) \otimes [\oplus_{m=1}^n L_\sigma^2 \Psi^n(X^m)]$. Thus, (3.18) yields the statement. □

We give also a Fock space representation of the operator $H_{\pi_\sigma}^{\mathrm{B}}$. Corollary 2.1 implies the following:

Corollary 3.1. *Let the conditions of Theorem* 3.4(2) *be satisfied. Then*,

$$\mathcal{I}^n H_{\pi_\sigma}^{\mathrm{B}} (\mathcal{I}^n)^{-1} = \mathrm{d}\,\mathrm{Exp}\, H_\sigma \boxplus \left(\bigoplus_{m=1}^n H_{\sigma,(n,m)}^{\mathrm{B}} \right),$$

cf. Remark 3.1.

3.4. de Rham Laplacian on forms over the configuration space

We define linear operators

$$d^\Gamma : \mathcal{F}\Omega^n \to \mathcal{F}\Omega^{n+1}, \quad n \in \mathbb{N}_0, \quad \mathcal{F}\Omega^0 := \mathcal{FC}, \tag{3.27}$$

by

$$(d^\Gamma W)(\gamma) := (n+1)^{1/2} \mathrm{AS}_{n+1}(\nabla^\Gamma W(\gamma)), \tag{3.28}$$

where $\mathrm{AS}_{n+1} : (T_\gamma \Gamma_X)^{\otimes(n+1)} \to \wedge^{n+1}(T_\gamma \Gamma_X)$ is the antisymmetrization operator. It follows from this definition that

$$(d^\Gamma W)(\gamma) = \sum_{x \in \gamma} (d_x^X W)(\gamma), \tag{3.29}$$

where

$$(d_x^X W)(\gamma) := \sum_{[x_1,\dots,x_n]_d \subset \gamma} d^X (W_x(\gamma, x)_{[x_1,\dots,x_n]_d})$$
$$= \sum_{[x_1,\dots,x_n]_d \subset \gamma} (n+1)^{1/2}\, \mathrm{AS}_{n+1}(\nabla^X W_x(\gamma, x)_{[x_1,\dots,x_n]_d}) \tag{3.30}$$

with $\mathrm{AS}_{n+1} : T_x X \otimes (T_{x_1} X \wedge \cdots \wedge T_{x_n} X) \to T_x X \wedge T_{x_1} X \wedge \cdots \wedge T_{x_n} X$ being again the antisymmetrization. Therefore, we have indeed the inclusion $d^\Gamma \omega \in \mathcal{F}\Omega^{n+1}$ for each $\omega \in \mathcal{F}\Omega^n$.

Suppose that, in local coordinates on the manifold X, the form $W_x(\gamma, \bullet)_{[x_1,\dots,x_n]_d}$ has the representation

$$\mathcal{O}_{\gamma,x} \ni y \mapsto W_x(\gamma, y)_{[x_1,\dots,x_n]_d} = w(y)\, h_1 \wedge \cdots \wedge h_n, \tag{3.31}$$

where $w : \mathcal{O}_{\gamma,x} \to \mathbb{R}$ and $h_1 \wedge \cdots \wedge h_n \in T_{x_1} \wedge \cdots \wedge T_{x_n}$. Then,

$$\mathrm{AS}_{n+1}(\nabla^X W_x(\gamma, x)_{[x_1,\dots,x_n]_d}) = \nabla^X w(x) \wedge h_1 \wedge \cdots \wedge h_n, \tag{3.32}$$

which, upon (3.30), describes the action of d_x^X.

Let us consider now d^Γ as an operator acting from the space $L^2_{\pi_\sigma}\Omega^n$ into $L^2_{\pi_\sigma}\Omega^{n+1}$. Analogously to the proof of Theorem 3.3, we get the following formula for the adjoint operator $d^{\Gamma *}_{\pi_\sigma}$ restricted to $\mathcal{F}\Omega^{n+1}$:

$$(d^{\Gamma *}_{\pi_\sigma} W)(\gamma) = \sum_{x \in \gamma} (d^{X*}_{\sigma,x} W)(\gamma), \tag{3.33}$$

where

$$(d^{X*}_{\sigma,x}) W(\gamma) = \sum_{[x_1,\dots,x_{n+1}]_d \subset \gamma : x \in \{x_1,\dots,x_{n+1}\}} d^{X*}_{\sigma,x}(W_x(\gamma, x)_{[x_1,\dots,x_{n+1}]_d}). \tag{3.34}$$

Suppose, analogously to the above, that in local coordinates on the manifold X

$$\mathcal{O}_{\gamma,x} \ni y \mapsto W_x(\gamma, y)_{[x_1,\dots,x_{n+1}]_d} = w(y) h_1 \wedge \cdots \wedge h_{n+1}, \tag{3.35}$$

where $w : \mathcal{O}_{\gamma,x} \to \mathbb{R}$ and $h_1 \wedge \cdots \wedge h_{n+1} \in T_{x_1} X \wedge \cdots \wedge T_{x_{n+1}} X$. Then,

$$d^{X*}_{\sigma,x}(W_x(\gamma, x)_{[x_1,\dots,x_{n+1}]_d}) = -(n+1)^{-1/2} \sum_{i=1}^{n+1} (-1)^{i-1} \delta_{x,x_i} [\langle \nabla^X w(x), h_i \rangle_x$$
$$+ w(x) \langle \beta_\sigma(x), h_i \rangle_x] h_1 \wedge \cdots \wedge \check{h}_i \wedge \cdots \wedge h_{n+1}. \tag{3.36}$$

Here,

$$\delta_{x,x_i} = \begin{cases} 1 & \text{if } x = x_i, \\ 0 & \text{otherwise,} \end{cases}$$

and $\check{h}_i$ denotes the absence of h_i.

Upon (3.33)–(3.36),

$$d_{\pi_\sigma}^{\Gamma *}: \mathcal{F}\Omega^{n+1} \to L^2_{\pi_\sigma}\Omega^n.$$

For $n \in \mathbb{N}$, we define the pre-Dirichlet form $\mathcal{E}^{\mathrm{R}}_{\pi_\sigma}$ by

$$\mathcal{E}^{\mathrm{R}}_{\pi_\sigma}(W^{(1)}, W^{(2)}) := \int_{\Gamma_X} [\langle d^\Gamma W^{(1)}(\gamma), d^\Gamma W^{(2)}(\gamma)\rangle_{\wedge^{n+1}(T_\gamma \Gamma_X)} + \langle d^{\Gamma *}_{\pi_\sigma} W^{(1)}(\gamma), d^{\Gamma *}_{\pi_\sigma} W^{(2)}(\gamma)\rangle_{\wedge^{n-1}(T_\gamma \Gamma_X)}]\pi_\sigma(\mathrm{d}\gamma), \quad (3.37)$$

where $W^{(1)}, W^{(2)} \in \mathcal{F}\Omega^n$. Analogously to the case of Bochner, we conclude that the function under the sign of integral in (3.37) is polynomially bounded, so that the integral exists.

The next theorem follows from (3.28)–(3.35) and (3.36).

Theorem 3.5. *For any $W^{(1)}, W^{(2)} \in \mathcal{F}\Omega^n$, we have*

$$\mathcal{E}^{\mathrm{R}}_{\pi_\sigma}(W^{(1)}, W^{(2)}) = \int_{\Gamma_X} \langle H^{\mathrm{R}}_{\pi_\sigma} W^{(1)}(\gamma), W^{(2)}\rangle_{\wedge^n(T\Gamma_X)}\pi_\sigma(\mathrm{d}\gamma).$$

Here, $H^{\mathrm{R}}_{\pi_\sigma} = d^\Gamma d^{\Gamma *}_{\pi_\sigma} + d^{\Gamma *}_{\pi_\sigma} d$ is an operator in the space $L^2_{\pi_\sigma}\Omega^n$ with domain $\mathcal{F}\Omega^n$. It can be represented as follows:

$$H^{\mathrm{R}}_{\pi_\sigma} W(\gamma) = \sum_{x\in\gamma} H^{\mathrm{R}}_{\sigma,x} W(\gamma) = \langle H^{\mathrm{R}}_{\sigma,\bullet} W(\gamma), \gamma\rangle, \quad W \in \mathcal{F}\Omega^n, \quad (3.38)$$

where

$$H^{\mathrm{R}}_{\sigma,x} = d^X_x d^{X*}_{\sigma,x} + d^{X*}_{\sigma,x} d^X_x. \quad (3.39)$$

From Theorem 3.5, we conclude that the pre-Dirichlet form $\mathcal{E}^{\mathrm{R}}_{\pi_\sigma}$ is closable in the space $L^2_{\pi_\sigma}\Omega^n$. The generator of its closure (being actually the Friedrichs extension of the operator $H^{\mathrm{R}}_{\pi_\sigma}$, for which we preserve the same notation) will be called the de Rham Laplacian on Γ_X corresponding to the Poisson measure π_σ. By (3.38) and (3.39), $H^{\mathrm{R}}_{\pi_\sigma}$ is the lifting of the de Rham Laplacian on X with measure σ.

Remark 3.6. *Similarly to* (3.17), *the operator $H^{\mathrm{R}}_{\pi_\sigma}$ preserves the space $\mathcal{F}\Omega^n$, and we can always take $\Lambda(H^{\mathrm{R}}_{\pi_\sigma} W) = \Lambda(W)$. Then, for any open bounded $\Lambda \supset \Lambda(W)$, we have*

$$(H^{\mathrm{R}}_{\pi_\sigma} W)_{\Lambda,\gamma} = H^{\mathrm{R}}_{\sigma^{\otimes|\Lambda\cap\gamma|}}(X^{|\Lambda\cap\gamma|})W_{\Lambda,\gamma}, \quad (3.40)$$

where $H^{\mathrm{B}}_{\sigma^{\otimes|\Lambda\cap\gamma|}}(X^{|\Lambda\cap\gamma|})$ is the de Rham Laplacian of the manifold $X^{|\Lambda\cap\gamma|}$ with the product measure $\sigma^{\otimes|\Lambda\cap\gamma|}$.

Analogously to Theorem 3.4, we get the following theorem.

Theorem 3.6.

1. *On $\mathcal{D}\Omega^n$ we have*

$$H^{\mathrm{R}}_{\pi_\sigma} = (I^n)^{-1}\left[H_{\pi_\sigma} \boxplus \left(\bigoplus_{m=1}^{n} H^{\mathrm{R}}_{\sigma,(n,m)}\right)\right] I^n, \tag{3.41}$$

   where $H^{\mathrm{R}}_{\sigma,(n,m)}$ denotes the restriction of the de Rham Laplacian acting in the space $L^2(X^m \to \wedge^n(TX^m); \sigma^{\otimes m})$ to the subspace $L^2_\sigma \Psi^n(X^m)$.

2. *Suppose that, for each $m = 1, \dots, n$, the de Rham Laplacian $H^{\mathrm{R}}_{\sigma,m}(X)$ is essentially self-adjoint on the set of smooth forms with compact support. Then, $\mathcal{D}\Omega^n$ is a domain of essential self-adjointness of $H^{\mathrm{R}}_{\pi_\sigma}$, and the equality* (3.41) *holds for the closed operators $H^{\mathrm{R}}_{\pi_\sigma}$ and $H_{\pi_\sigma} \boxplus (\oplus_{m=1}^{n} H^{\mathrm{R}}_{\sigma,(n,m)})$, where the latter operator is closed from its domain of essential self-adjointness $I^n(\mathcal{D}\Omega^n)$.*

Remark 3.7. *The essential self-adjointness of the de Rham Laplacian H^{R}_σ on the set of smooth forms with compact support is well known in the case where σ is the volume measure* (*see, e.g.,* [20]). *It is also sufficient to assume that β_σ, together with its derivatives up to order* 3, *as well as the curvature tensor of X, together with its derivatives up to order* 2, *are bounded* (*cf. Remark* 3.5).

Proof of Theorem 3.6.

1. Upon (3.7), (3.8), (3.27)–(3.36), (3.38) and (3.39), we get, for any $W \in \mathcal{D}\Omega^n$ given by the formula (2.23),

$$(H^{\mathrm{R}}_{\pi_\sigma} W)_k(\gamma)(\bar{x}) = 0 \quad \text{for } k \neq m,$$

$$(H^{\mathrm{R}}_{\sigma,x} W)_m(\gamma)(\bar{x}) = \begin{cases} (m!)^{1/2}(H_{\sigma,x} F)(\gamma \setminus \{\bar{x}\})\omega(\bar{x}), & x \in \gamma \setminus \{\bar{x}\}, \\ (m!)^{1/2} F(\gamma \setminus \{\bar{x}\})(H^{\mathrm{R}}_{\sigma,x}\omega)(\bar{x}), & x \in \{\bar{x}\}. \end{cases}$$

   Hence, analogously to (3.19) and (3.20), we derive

$$(I^n_k H^{\mathrm{R}}_{\pi_\sigma} W)(\gamma, \bar{x}) = \begin{cases} 0, & k \neq m, \\ (H_{\pi_\sigma} F)(\gamma)\omega(\bar{x}) + F(\gamma)(H^{\mathrm{R}}_{\sigma,(n,m)}\omega)(\bar{x}), & k = m, \end{cases}$$

   which easily yields (3.41).

2. The proof is similar to that of Theorem 3.4(2). □

Again, analogously to Corollary 3.1, we get a Fock space representation of the operator $H^{\mathrm{R}}_{\pi_\sigma}$.

Corollary 3.2. *Let the conditions of Theorem* 3.6(2) *be satisfied. Then,*

$$\mathcal{I}^n H^{\mathrm{R}}_{\pi_\sigma} (\mathcal{I}^n)^{-1} = \mathrm{d}\,\mathrm{Exp}\, H_\sigma \boxplus \left(\bigoplus_{m=1}^{n} H^{\mathrm{R}}_{\sigma,(n,m)}\right).$$

3.5. Weitzenböck formula on the configuration space

In this section, we will derive a generalization of the Weitzenböck formula to the case of the Poisson measure on the configuration space. In other words, we will derive a formula which gives a relation between the Bochner and de Rham Laplacians.

Analogously to (3.7) and (3.8), we define for each $V(\gamma) \in T_\gamma \Gamma_X$, $\gamma \in \Gamma_X$, the annihilation and creation operators

$$a(V(\gamma)) : \wedge^{n+1}(T_\gamma \Gamma_X) \to \wedge^n(T_\gamma \Gamma_X), \qquad a^*(V(\gamma)) : \wedge^n(T_\gamma \Gamma_X) \to \wedge^{n+1}(T_\gamma \Gamma_X)$$

as follows:

$$a(V(\gamma))W_{n+1}(\gamma) = (n+1)^{1/2}\langle V(\gamma), W_{n+1}(\gamma)\rangle_\gamma, \qquad W_{n+1}(\gamma) \in \wedge^{n+1}(T_\gamma \Gamma_X),$$
$$a^*(V(\gamma))W_n(\gamma) = (n+1)^{1/2} V(\gamma) \wedge W_n(\gamma), \qquad W_n(\gamma) \in \wedge^n(T_\gamma \Gamma_X).$$

Now, for a fixed $\gamma \in \Gamma_X$ and $x \in \gamma$, we define the operator $R(\gamma)$ as follows:

$$R(\gamma) = \sum_{x\in\gamma} R(\gamma, x), \qquad D(R(\gamma)) := \wedge_0^n(T_\gamma \Gamma_X),$$
$$R(\gamma, x) := \sum_{i,j,k,l=1}^{d} R_{ijkl}(x) a^*(e_i) a(e_j) a^*(e_k) a(e_l).$$

Here, $\{e_j\}_{j=1}^d$ is a fixed orthonormal basis in the space $T_x X$ considered as a subspace of $T_\gamma \Gamma_X$, and $\wedge_0^n(T_\gamma \Gamma_X)$ consists of all $W(\gamma) \in \wedge^n(T_\gamma \Gamma_X)$ having only a finite number of nonzero coordinates in the direct sum expansion (2.5).

Next, we note that

$$\begin{aligned}\nabla^\Gamma B_{\pi_\sigma}(\gamma) &= (\nabla_x^X B_{\pi_\sigma}(\gamma))_{x\in\gamma} = (\nabla_x^X (B_{\pi_\sigma}(\gamma)_y))_{x,y\in\gamma} \\ &= (\delta_{x,y} \nabla^X \beta_\sigma(y))_{x,y\in\gamma} \in (T_{\gamma,\infty}\Gamma_X)^{\otimes 2}.\end{aligned}$$

Hence, for any $V(\gamma) \in T_{\gamma,0}\Gamma_X$,

$$\begin{aligned}\nabla_V^\Gamma B_{\pi_\sigma}(\gamma) := \langle \nabla^\Gamma B_{\pi_\sigma}(\gamma), V(\gamma)\rangle_\gamma &= \left(\sum_{y\in\gamma} \delta_{x,y} \langle \nabla^X \beta_\sigma(y), V(\gamma)_y\rangle_y\right)_{x\in\gamma} \\ &= (\langle \nabla^X \beta_\sigma(x), V(\gamma)_x\rangle_x)_{x\in\gamma} \in T_{\gamma,0}\Gamma_X.\end{aligned}$$

Thus, $\nabla^\Gamma B_{\pi_\sigma}(\gamma)$ determines the linear operator in $T_{\gamma,0}\Gamma_X$ given by

$$T_{\gamma,0}\Gamma_X \ni V(\gamma) \mapsto \nabla^\Gamma B_{\pi_\sigma}(\gamma) V(\gamma) := \nabla_V^\Gamma B_{\pi_\sigma}(\gamma) \in T_{\gamma,0}\Gamma_X.$$

Analogously to (3.9), we define in $\wedge_0^n(T_\gamma \Gamma_X)$ the operator

$$\begin{aligned}(\nabla^\Gamma B_{\pi_\sigma}(\gamma))^{\wedge n} :=\; &\nabla^\Gamma B_{\pi_\sigma}(\gamma) \otimes \mathbf{1} \cdots \otimes \mathbf{1} + \mathbf{1} \otimes \nabla^\Gamma B_{\pi_\sigma}(\gamma) \otimes \mathbf{1} \otimes \cdots \otimes \mathbf{1} \\ &+ \cdots + \mathbf{1} \otimes \cdots \otimes \mathbf{1} \otimes \nabla^\Gamma B_{\pi_\sigma}(\gamma).\end{aligned}$$

Theorem 3.7 (Weitzenböck formula on the Poisson space). *We have on* $\mathcal{F}\Omega^n$

$$H^{\mathrm{R}}_{\pi_\sigma} = H^{\mathrm{B}}_{\pi_\sigma} + R_{\pi_\sigma}(\gamma), \tag{3.42}$$

where

$$R_{\pi_\sigma}(\gamma) := R(\gamma) - (\nabla^\Gamma B_{\pi_\sigma}(\gamma))^{\wedge n}. \tag{3.43}$$

Proof. Fix $W \in \mathcal{F}\Omega^n$ and $\gamma \in \Gamma_X$. Let $\Lambda(W) \subset X$ be a compactum as in Definition 2.3 corresponding to W, and let Λ be an open set in X with compact closure such that $\Lambda(W) \subset \Lambda$. Next, let $W_{\Lambda,\gamma}$ be the form on $\mathcal{O}_{\gamma,x_1} \times \cdots \times \mathcal{O}_{\gamma,x_k}$, $\{x_1, \dots, x_k\} = \gamma \cap \Lambda$, defined by (2.10).

It follows from Remarks 3.4(2) and 3.6 that

$$\begin{aligned}
&\mathrm{Proj}_{\wedge^n(T_{x_1}\oplus\cdots\oplus T_{x_k})}(H^{\mathrm{B}}_{\pi_\sigma} W(\gamma)) = H^{\mathrm{B}}_{\sigma^{\otimes|\Lambda\cap\gamma|}}(X^{|\Lambda\cap\gamma|}) W_{\Lambda,\gamma}(x_1, \dots, x_k),\\
&\mathrm{Proj}_{\wedge^n(T_{x_1}\oplus\cdots\oplus T_{x_k})}(H^{\mathrm{R}}_{\pi_\sigma} W(\gamma)) = H^{\mathrm{R}}_{\sigma^{\otimes|\Lambda\cap\gamma|}}(X^{|\Lambda\cap\gamma|}) W_{\Lambda,\gamma}(x_1, \dots, x_k),
\end{aligned}$$

and $H^{\mathrm{B}}_{\pi_\sigma} W(\gamma)_{[y_1,\dots,y_n]_d} = H^{\mathrm{R}}_{\pi_\sigma} W(\gamma)_{[y_1,\dots,y_n]_d} = 0$, $[y_1, \dots, y_n]_d \subset \gamma$, if at least one $y_i \in \{y_1, \dots, y_n\}$ does not belong to Λ. Now, the formulae (3.42) and (3.43) follow from the usual Weitzenböck formulae (3.10) and (3.11) for the operators $H^{\mathrm{B}}_{\sigma^{\otimes|\Lambda\cap\gamma|}}(X^{|\Lambda\cap\gamma|})$ and $H^{\mathrm{R}}_{\sigma^{\otimes|\Lambda\cap\gamma|}}(X^{|\Lambda\cap\gamma|})$. □

We will show now that the Weitzenböck correction term R_{π_σ} is a lifting of the Weitzenböck correction terms $R_{\sigma,k}$ of the manifold X.

Given operator fields

$$X \ni x \mapsto J_k(x) \in \mathcal{L}(\wedge^k(T_x X)), \quad k = 1, \dots, \min\{n, d\}, \tag{3.44}$$

which are supposed to be uniformly bounded, we define a "diagonal" operator field

$$\tilde{X}^m \ni \bar{x} \mapsto J_{n,m}(\bar{x}) \in \mathcal{L}(\mathbb{T}^{(n)}_{\{\bar{x}\}} X^m), \quad m = 1, \dots, n, \tag{3.45}$$

as follows. First, we define for each $\bar{x} = (x_1, \dots, x_m) \in \tilde{X}^m$ operators

$$\begin{aligned}
&J^{k_1,\dots,k_m}_{n,m}(x_1, \dots, x_m) \in \mathcal{L}((T_{x_1}X)^{\wedge k_1} \wedge \cdots \wedge (T_{x_m}X)^{k_m}),\\
&\quad 1 \le k_1, \dots, k_m \le d, \quad k_1 + \cdots + k_m = n,
\end{aligned}$$

by setting

$$\begin{aligned}
&J^{k_1,\dots,k_m}_{n,m}(x_1, \dots, x_m) u_1^{(k_1)} \wedge \cdots \wedge u_m^{(k_m)} := (J_{k_1}(x_1) u_1^{(k_1)}) \wedge u_2^{(k_2)} \wedge \cdots \wedge u_m^{(k_m)}\\
&\quad + u_1^{(k_1)} \wedge (J_{k_2}(x_2) u_2^{(k_2)}) \wedge \cdots \wedge u_m^{(k_m)} + \cdots\\
&\quad + u_1^{(k_1)} \wedge \cdots \wedge u_{m-1}^{(k_{m-1})} \wedge (J_{k_m}(x_m) u_m^{(k_m)}),\\
&\quad u_i^{(k_i)} \in \wedge^{k_i}(T_{x_i}X), \quad i = 1, \dots, m,
\end{aligned} \tag{3.46}$$

and extending the operator $J_{n,m}^{k_1,\dots,k_m}(x_1,\dots,x_m)$ by linearity and continuity to the whole space. Then, the operator $J_{n,m}(x_1,\dots,x_m) \in \mathcal{L}(\mathbb{T}^{(n)}_{\{x_1,\dots,x_m\}}X^m)$ is defined by setting its diagonal blocks in the decomposition (2.16) of the space $\mathbb{T}^{(n)}_{\{x_1,\dots,x_m\}}X^m$ to be $J_n^{k_1,\dots,k_m}(x_1,\dots,x_m)$ and the other blocks to be equal to zero.

Notice that, for each $\nu \in S_m$, the operators $J_{n,m}(x_1,\dots,x_m)$ and $J_{n,m}(x_{\nu(1)},\dots,x_{\nu(m)})$ coincide, so that (3.45) naturally determines the operator field

$$\tilde{X}^m/S_m \ni \{\bar{x}\} \mapsto J_{n,m}(\{\bar{x}\}) \in \mathcal{L}(\mathbb{T}^{(n)}_{\{\bar{x}\}}X^m), \quad m = 1,\dots,n. \tag{3.47}$$

Now, we define an operator field

$$\Gamma_X \ni \gamma \mapsto \mathbf{J}(\gamma) \in \mathcal{L}(\wedge^n(T_\gamma \Gamma_X)) \tag{3.48}$$

setting $\mathbf{J}(\gamma)$ to be again the block-diagonal operator in the decomposition (2.17) with the diagonal blocks $J_{n,m}(\{\bar{x}\})$ and the other blocks equal to zero.

In what follows, we suppose, for simplicity, that

$$\text{the curvature tensor } R_{ijkl}(x) \text{ and } \nabla^X \beta_\sigma(x) \text{ are uniformly bounded in } x \in X. \tag{3.49}$$

As easily seen, for each $k \in \mathbb{N}$, the Weitzenböck correction term $R_{\sigma,k}(\bullet)$ on the manifold X is now a uniformly bounded operator field taking values in $\wedge^k(TX)$ (cf. (3.11)). Thus, we can define an operator field $\mathbf{R}_\sigma(\gamma)$ through the operator fields $R_{\sigma,k}(x)$.

Proposition 3.1. *Let* (3.49) *hold. Then,*

$$R_{\pi_\sigma} = \mathbf{R}_\sigma.$$

Proof. The result can be easily seen directly from the definition of $\mathbf{R}_\sigma(\gamma)$, $R(\gamma)$ and $B_{\pi_\sigma}(\gamma)$. □

4. Probabilistic representation of the Laplacians

Let $\xi_x(t)$ be the Brownian motion on X with the drift β_σ, the logarithmic derivative of σ, which starts at a point $x \in X$. We suppose the following:

- for each $x \in X$, the process $\xi_x(t)$ has infinite life-time;
- the semigroup

  $$T_0(t)f(x) := \mathsf{E} f(\xi_x(t))$$

  preserves the space $C_b^2(X)$ and can be extended to a strongly continuous semigroup of contractions in $L^2(X;\sigma)$, and its generator H_0 is essentially self-adjoint on the space $\mathcal{D}$ (in this case $H_0 = H_\sigma$).

Remark 4.1. *The above conditions are fulfilled if, e.g.,* β_σ*, together with its derivatives up to order* 3*, is bounded.*

We denote by $\xi_\gamma(t)$ the corresponding independent infinite particle process which starts at a point $\gamma \in \Gamma_X$,

$$\xi_\gamma(t) = (\xi_x(t))_{x\in\gamma}.$$

As we already mentioned in Section 3.1, this process is properly associated with the Dirichlet form $\mathcal{E}_{\pi_\sigma}$ (see [8]).

Remark 4.2. *The process ξ_γ lives in general on the bigger state space $\ddot{\Gamma}$ consisting of all $\mathbb{Z}_+$-valued Radon measures on X* (*the space $\ddot{\Gamma}_X$ being Polish*). *Notice, however, that at each fixed moment of time $t \in \mathbb{R}_+$ the value $\xi_\gamma(t)$ belongs to Γ_X a.s. Moreover, it was proven in* [34] *that, in the special case $X = \mathbb{R}^d$ with $d \geq 2$, the process ξ_γ lives a.s. in Γ_X.*

Let $\mathbf{T}_0(t)F(\gamma) := \mathsf{E}F(\xi_\gamma(t))$ be the corresponding semigroup. As shown in [8], it can be extended from $\mathcal{F}C_{\mathrm{b}}^\infty(\Gamma_X)$ to a strongly continuous semigroup in $L^2_{\pi_\sigma}(\Gamma_X)$ with the generator $\mathbf{H}_0 = H^\Gamma_{\pi_\sigma}$.

Given operator fields (3.44) which are now supposed to be uniformly bounded, continuous, and symmetric, we define again operator fields (3.45) in the same way as in Section 3.5. We have obviously $J_{n,m}(\bar{x})^* = J_{n,m}(\bar{x})$.

Let

$$P^{J_{n,m}}_{\xi_{\bar{x}}}(t) : \mathbb{T}^{(n)}_{\{\bar{x}\}}X^m \to \mathbb{T}^{(n)}_{\{\xi_{\bar{x}}(t)\}}X^m, \quad m = 1, \dots, n,$$

be the parallel translation along the path $\xi_{\bar{x}}(t) := (\xi_{x_i}(t))_{i=1,\dots,m}$ with the potential $J_{n,m}$. That is, $\eta(t) = P^{J_{n,m}}_{\xi_{\bar{x}}}(t)h$ satisfies the SDE

$$\frac{\mathrm{D}}{\mathrm{d}t}\eta(t) = J_{n,m}(\eta(t)), \qquad \eta(0) = h, \tag{4.1}$$

where $D/\mathrm{d}t$ denotes the covariant differentiation along the paths of the process ξ (see [20]). It is easy to see that the symmetry of the potential $J_{n,m}(\bar{x})$ with respect to a permutation of the components of $\bar{x}$ implies the same symmetry of $P^{J_{n,m}}_{\xi_{\bar{x}}}(t)$. Thus, analogously to (3.47), we get the operator field

$$\tilde{X}^m/S_m \ni \{\bar{x}\} \mapsto P^{J_{n,m}}_{\{\xi_{\bar{x}}\}}(t). \tag{4.2}$$

Now, for π_σ-a.e. $\gamma \in \Gamma_X$, we define the operator

$$\mathbf{P}^{\mathbf{J}}_{\xi_\gamma}(t) : \wedge^n(T_\gamma \Gamma_X) \to \wedge^n(T_{\xi_\gamma(t)}\Gamma_X)$$

by setting its diagonal blocks in the decomposition (2.17) to be $P^{J_{n,m}}_{\{\xi_{\bar{x}}\}}(t)$ and the other blocks to be equal to zero.

It is known that

$$\|P^{J_{n,m}}_{\xi_{\bar{x}}}(t)\| \leq \mathrm{e}^{tC_m}, \quad m = 1, \dots, n, \tag{4.3}$$

where C_m is the supremum of the spectrum of $J_{n,m}(\bar{x})$.

Lemma 4.1. *For π_σ-a.e. $\gamma \in \Gamma_X$, we have*

$$\|\mathbf{P}^{\mathbf{J}}_{\xi_\gamma}(t)\| \le \mathrm{e}^{tC}, \qquad C = \max_{m=1,\dots,n} C_m. \tag{4.4}$$

Proof. The result follows directly from the definition of $\mathbf{P}^{\mathbf{J}}_{\xi_\gamma}(t)$ and estimate (4.3). □

Let us define a semigroup $\mathbf{T}^{\mathbf{J}}_n(t)$ acting in the space of n-forms as follows:

$$\mathbf{T}^{\mathbf{J}}_n(t)W(\gamma) := \mathsf{E}(\mathbf{P}^{\mathbf{J}}_{\xi_\gamma}(t))^* W(\xi_\gamma(t)), \qquad W \in \mathcal{F}\Omega^n. \tag{4.5}$$

Let $T^J_{n,m}(t)$ be the semigroup acting in the space $L^2_\sigma \Psi^n(X^m)$ as

$$T^J_{n,m}(t)\omega(\bar{x}) := \mathsf{E}(P^{J_{n,m}}_{\xi_{\bar{x}}}(t))^*\omega(\xi_{\bar{x}}(t)). \tag{4.6}$$

By virtue of (4.2) and estimate (4.3), we conclude the correctness of the definition of $T^J_{n,m}(t)$ (in the sense that $T^J_{n,m}(t)$ is uniquely defined) and its strong continuity. The following result describes the structure and properties of the semigroup $\mathbf{T}^{\mathbf{J}}_n(t)$.

Proposition 4.1.
1. $\mathbf{T}^{\mathbf{J}}_n(t)$ *satisfies the estimate*

$$\|\mathbf{T}^{\mathbf{J}}_n(t)V(\gamma)\|_{\wedge^n(T_\gamma\Gamma_X)} \le \mathrm{e}^{tC}\,\mathbf{T}_0(t)\|V(\gamma)\|_{\wedge^n(T_\gamma\Gamma_X)} \tag{4.7}$$

for π_σ-a.e. $\gamma \in \Gamma_X$.

2. *Under the isomorphism I^n, $\mathbf{T}^{\mathbf{J}}_n(t)$ takes the following form*:

$$I^n_m \mathbf{T}^{\mathbf{J}}_n(t) = \mathbf{T}_0(t) \otimes T^J_{n,m}(t) I^n_m, \quad m = 1, \dots, n. \tag{4.8}$$

In particular, for 1-forms

$$I^1\mathbf{T}^{\mathbf{J}}_1(t) = \mathbf{T}_0(t) \otimes T^J_{1,1}(t) I^1. \tag{4.9}$$

3. $\mathbf{T}^{\mathbf{J}}_n(t)$ *extends to a strongly continuous semigroup in $L^2_{\pi_\sigma}\Omega^n$.*

Proof.
1. The result follows from formula (4.4).
2. For simplicity, we give the proof only in the case of 1-forms. Let $V \in \mathcal{D}\Omega^1$ be given by $I^1V = F \otimes v$. By the definition of $\mathbf{T}^{\mathbf{J}}_1(t)$ and the construction of the process ξ_γ, we have

$$\begin{aligned} \mathbf{T}^{\mathbf{J}}_1(t)V(\gamma)_x &= \mathsf{E}F(\xi_\gamma(t) \setminus \{\xi_x(t)\})(P^{J_{1,1}}_{\xi_x}(t))^* v(\xi_x(t)) \\ &= \mathsf{E}F(\xi_\gamma(t) \setminus \{\xi_x(t)\})\mathsf{E}_{\xi_x}(P^{J_{1,1}}_{\xi_x}(t))^* v(\xi_x(t)) \\ &= \mathbf{T}_0(t)F(\gamma \setminus \{x\})T^J_1(t)v(x), \end{aligned}$$

E_{ξ_x} meaning the expectation with respect to the process $\xi_x(t)$, from where the result follows. The general case can be proved by similar arguments.

3. The result follows from the corresponding results for semigroups $\mathbf{T}_0(t)$ and $T^J_{n,m}(t)$, which are well known (see [6], resp. [20]). □

Let $\mathbf{H}^{\mathbf{J}}_n$ and $H^J_{n,m}$ be the generators of $\mathbf{T}^{\mathbf{J}}_n(t)$ and $T^J_{n,m}(t)$, respectively.

Now, we will give probabilistic representations of the semigroups $T^{\mathrm{B}}_{\pi_\sigma}(t)$ and $T^{\mathrm{R}}_{\pi_\sigma}(t)$ associated with the operators $H^{\mathrm{B}}_{\pi_\sigma}$ and $H^{\mathrm{R}}_{\pi_\sigma}$, respectively. We set

$$J^{(1)}_m := 0, \qquad J^{(2)}_m(x) := R_{\sigma,m}(x), \quad m = 1, \dots, \min\{n, d\}$$

(cf. (3.11)). Let us remark that $P^{J^{(1)}_{n,m}}_{\xi_{\bar{x}}}(t) \equiv P_{\xi_{\bar{x}}}(t)$ is the parallel translation of the n-forms along the path $\xi_{\bar{x}}$, and we have

$$H^{J^{(1)}}_{n,m} = -H^{\mathrm{B}}_{\sigma,(n,m)}, \qquad H^{J^{(2)}}_{n,m} = -H^{\mathrm{R}}_{\sigma,(n,m)} \quad \text{on } \Psi^n_0(X^m).$$

Theorem 4.1.

1. *For $W \in \mathcal{D}\Omega^n$, we have*

$$H^{\mathrm{B}}_{\pi_\sigma} W = -\mathbf{H}^{\mathbf{J}^{(1)}}_n W, \qquad H^{\mathrm{R}}_{\pi_\sigma} W = -\mathbf{H}^{\mathbf{J}^{(2)}}_n W. \tag{4.10}$$

2. *As L^2-semigroups,*

$$T^{\mathrm{B}}_{\pi_\sigma}(t) = \mathbf{T}^{\mathbf{J}^{(1)}}_n(t), \qquad T^{\mathrm{R}}_{\pi_\sigma}(t) = \mathbf{T}^{\mathbf{J}^{(2)}}_n(t). \tag{4.11}$$

3. *The semigroups $T^{\mathrm{B}}_{\pi_\sigma}(t)$ and $T^{\mathrm{R}}_{\pi_\sigma}(t)$ satisfy the estimates*

$$\|T^{\mathrm{B}}_{\pi_\sigma}(t)V(\gamma)\|_\gamma \le \mathbf{T}_0(t)\|V(\gamma)\|_\gamma, \qquad \|T^{\mathrm{R}}_{\pi_\sigma}(t)V(\gamma)\|_\gamma \le \mathrm{e}^{tC}\,\mathbf{T}_0(t)\|V(\gamma)\|_\gamma$$

for π_σ-a.e. $\gamma \in \Gamma_X$.

Proof.

1. It follows directly from the decomposition (4.8) that, on $\mathcal{D}\Omega^n$, we have

$$I^n_m \mathbf{H}^{\mathbf{J}}_n = (\mathbf{H}_0 \boxplus H^J_{n,m}) I^n_m, \tag{4.12}$$

where $\mathbf{H}_0$ is the generator of $\mathbf{T}_0(t)$. Setting, respectively, $J_m := J^{(1)}_m$ and $J_m := J^{(2)}_m$ and comparing (3.18) with (4.12), we obtain the result.
2. The statement follows from (4.10) and the essential self-adjointness of $H^{\mathrm{B}}_{\pi_\sigma}$ and $H^{\mathrm{R}}_{\pi_\sigma}$ on $\mathcal{D}\Omega^n$ by applying Proposition 4.1(3) with $J_m = J^{(1)}_m$ and $J_m = J^{(2)}_m$.
3. The result follows from (4.7) and (4.11). □

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